



The effect of exogenous information on voluntary disclosure and market quality[☆]

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ABSTRACT

We analyze a model in which information may be voluntarily disclosed by a firm and/or by a third party, e.g., financial analysts. Due to its strategic nature, corporate voluntary disclosure is qualitatively different from third-party disclosure. Greater analyst coverage crowds out (crowds in) corporate voluntary disclosure when analysts mostly discover information that is available (unavailable) to the firm. Nevertheless, greater analyst coverage always improves the overall quality of public information. We base this claim on two market quality measures: price efficiency, which is statistical in nature, and liquidity, which is derived in a trading stage that follows the disclosure stage.

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1. Introduction

There is a growing literature on voluntary disclosure that studies how agents strategically decide whether to disclose or withhold their private information. Public companies, for example, are mandated to disclose certain in-

formation in their periodic reporting, but other information may be disclosed at the discretion of managers. For example, a firm does not have to disclose that a major customer is negotiating a deal with one of its competitors. Hence, corporate voluntary disclosure is often a major source of information in capital markets.¹ Another example is an entrepreneur who seeks funding from investors (venture capital funds, angels, etc.); the entrepreneur can choose whether to disclose or conceal the results of previous attempts to raise funding or to acquire new customers. Examples are not limited to financial markets. An incumbent politician may obtain private information about the

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¹ Beyer et al. (2010) find that approximately 66% of accounting-based return variance is generated by voluntary disclosures, 22% by analyst forecasts, 8% by earnings announcements, and 4% by SEC filings.

success or failure of policies she has supported, and can choose whether to disclose or conceal these results. In all of these examples, informed agents are reluctant to lie because of severe or even criminal punishment, or because once the information is voluntarily disclosed it can be easily verified. Instead, they can choose to withhold negative information, taking advantage of public uncertainty about whether they indeed have private information.

The current literature focuses on settings in which a single agent chooses whether to disclose or withhold private information and there is no other source that can potentially discover this information. In practice, various sources that may discover and reveal a firm's private information are very prevalent. For example, financial analysts and rating agencies provide additional information about public firms, investors can gather information through their social network about an entrepreneur, and the media and independent think tanks can assess public policies.

In this paper, we introduce such additional sources of information into a standard voluntary disclosure setting with uncertainty about information endowment. Our main question is how the possibility of information disclosure by a third party affects the aggregate amount of publicly available information. To answer this question, we first need to study the reaction of the disclosing agent to the possibility of a report by the third party and then to analyze the overall information that is revealed by both sources.

Our model departs from a standard voluntary disclosure setting with uncertain information endowment (à la Dye, 1985; Jung and Kwon, 1988). A manager of a public firm who wishes to maximize her firm's stock price may be endowed with private value-relevant information. The financial market prices the firm based on all publicly available information. If the manager is informed, she can credibly and costlessly disclose her information to the market. The novelty of our model is the additional external source of information, e.g., an analyst who may discover and publish information that the manager may privately hold. We assume that the analyst may discover and publish information when the manager is informed as well as when the manager is uninformed, and we allow for correlation in the manager's and analyst's information endowment.

We first show that, as is standard in this literature, the game has a unique equilibrium, in which the manager discloses the realization of her private information if and only if it is higher than an equilibrium threshold. We then study how the firm's disclosure strategy changes in response to an increase in analyst coverage, i.e., an increase in the probability that the analyst discovers and discloses information. We show that the directional effect depends on the information production of the analyst. If an increase in coverage affects more the probability of analysts to discover information that the manager is unaware of, e.g., information on market conditions, compared to information that she knows, then such an increase crowds in corporate voluntary disclosure. That is, firms respond to an increase in analyst coverage by decreasing the disclosure threshold, which increases the amount of disclosed information. If, however, an increase in coverage affects more the probability of analysts to discover information that the manager

knows, compared to information that she is unaware of, then such an increase crowds out corporate voluntary disclosure, that is, firms disclose less in response to an increase in analyst coverage. This last result, which is new to the theoretical literature, is consistent with the empirical evidence in Anantharaman and Zhang (2011), Balakrishnan et al. (2014), and Ellul and Panayides (2018) (see more details on the empirical literature below).

Given the empirical support for the crowding-out result, it is interesting to study the effect of an increase in analyst coverage on the overall amount of public information—including the information disclosed both by the analyst and by the manager. This is a challenging question, due to the qualitative difference between voluntary disclosure and information provided by analysts. While informed managers tend to disclose positive information and hide negative information, exogenous sources (such as analysts, the media, etc.) provide information that may be positive or negative. Thus, more exogenous information affects not only the *amount* of information that becomes available but also the *type* of information, and specifically the balance between positive and negative information. Formally, information in environments with varying levels of analyst coverage cannot be ranked using the Blackwell informativeness criterion.

We use two separate measures to capture the overall information available to the market. First, we consider a quadratic loss function, which equals the expected squared difference between the firm's actual and perceived value. This measure has a natural interpretation in terms of price efficiency or ex post return volatility. It can also represent the utility function of an information "receiver" such as an investor, and is consistent with the assumption that such a receiver sets prices to be equal to the expected value, conditional on all available information.

Our second measure of information quality is more specific to the capital market example and can be directly linked to empirical findings. We use the expected bid-ask spread as a measure that reflects the extent of information asymmetry in the market. We augment the disclosure model by introducing a trading stage à la Glosten and Milgrom (1985) that follows the disclosure stage. The trade and pricing in this stage are affected by the information that was revealed by the manager and the analyst. Our main result is that both price efficiency and liquidity increase as a result of an increase in analyst coverage; that is, the overall effect of an increase in third-party disclosure on market quality is always positive.

The economic argument for our result relies on the qualitative difference between the two information sources: the analyst and the firm. While an increase in the probability that the analyst reports affects pricing of all types of firms (good and bad), changes in voluntary disclosure affect only those types that are close to the disclosure threshold, that is, those that their pricing after they cease to be disclosed is close to the pricing they obtain following disclosure. Thus, the quality of public information is more sensitive to the former than to the latter.

Another way to put the intuition is that an increase in analyst coverage changes the balance between the negative and the positive information that is being disclosed:

more negative information is now being disclosed, while the positive information is less affected (since it is disclosed by the firm as well as the analyst). Thus, the overall quality of information improves. The change in the balance between negative and positive information, due to an increase in third-party disclosure, should be reflected in the skewness of returns. While firms with little coverage will exhibit strong positive skewness of the disclosed information, an increase in analyst coverage should make the distribution of public information more symmetric. Support for this can be found in [Acharya et al. \(2011\)](#), who find that larger firms exhibit a more symmetric return distribution. This seems to be consistent with our findings, since smaller firms receive less attention from exogenous information sources such as financial analysts and the media.

To complete our analysis, we analyze an additional channel through which analyst coverage can affect voluntary disclosure. It seems natural to believe that, if the analyst issues a report and the firm does not disclose, the manager may be accused of hiding unfavorable information and incur reputation or litigation costs. We extend the basic model by introducing such a cost, which increases in the belief of investors that the manager is actively withholding information. We show that this cost does result in more disclosure, though it cannot generate full disclosure. Thus, analyst coverage may also have a disciplinary role by incentivizing the manager to precautionarily disclose information. We further show that the effect of analyst coverage on precautionary disclosure is ambiguous, and depends on the analyst's information production: if the analyst is relatively more likely to uncover information that is already available to the manager, then an analyst report is a stronger signal that the manager is withholding unfavorable information, and an increase in analyst coverage increases precautionary disclosure.

Finally, we discuss the case where an informed analyst's signal is less precise than the signal of an informed manager. This assumption is realistic for various types of private information, e.g., a firm's internal information. We show that our results continue to hold given some additional assumptions on the information production technology of the analyst.

Our results can be used to assess certain policies that aim to increase market transparency in settings with voluntary disclosure. Such policies often focus on improving the information provided by one market participant without considering its effect on other market participants and the overall information available to the market.² Financial analysts have an important role in revealing firms' private information to the capital market, but there are other sources of exogenous revelation, such as news media, social media, competitors, suppliers, and the govern-

² Examples of regulations that focus on information provision include: the Sarbanes-Oxley Act, which attempts to increase the mandated reporting of firms; the Williams Act of 1968, which limits the ability of investors to trade anonymously on their private (optimistic) information; the regulation on analyst certification (Reg AC), which requires analysts to disclose possible conflicts of interest and prevent biased reports; the Dodd-Frank Act, which includes several measures aimed at improving the transparency and viability of credit ratings. See also the discussion in [Goldstein and Yang \(2017\)](#).

ment. Our results show that an improvement in one information source may crowd out information from another source, and that different parties affect the overall amount of public information differently. Our model suggests that to the extent that increasing the likelihood of such information discovery is not too costly, it is beneficial in terms of price efficiency and liquidity.

Unlike the theoretical literature (which is reviewed in the next subsection), the empirical literature studies the effect of analyst coverage on firms' voluntary disclosure and on the liquidity of firms' stock. Empirical evidence supports the predictions of our model. For example, [Kelly and Ljungqvist \(2012\)](#) show that following an exogenous decrease in analyst coverage, due to mergers and closings in the brokerage industry, information about affected firms became more asymmetric, and the liquidity of these firms' stocks decreased. [Ellul and Panayides \(2018\)](#) use a statistical model to identify exogenous terminations of analyst coverage. They show that stocks of firms that have lost complete analyst coverage experience a decrease in both liquidity and price efficiency.³

[Anantharaman and Zhang \(2011\)](#) and [Balakrishnan et al. \(2014\)](#) use the same exogenous negative shock to analyst coverage that is used in [Kelly and Ljungqvist \(2012\)](#) to establish the effect of a decrease in analyst coverage on firms' voluntary disclosure. [Balakrishnan et al. \(2014\)](#) show that one quarter following the decrease in analyst coverage, the affected firms increased their voluntary disclosure (earning guidance) to mitigate the increase in information asymmetry and the decrease in liquidity. This increased disclosure partially reverses the decrease in liquidity, although the overall effect remains negative, consistent with both predictions of our model. [Ellul and Panayides \(2018\)](#) divide their sample of firms that experienced unexpected coverage termination into those that increased the number of news releases in the post-termination period and those that kept the number unchanged or decreased it. They show that liquidity deteriorates less in the former group, suggesting again that firms disseminate more information to the market to mitigate the effect of the decrease in analyst coverage.

Following a review of the related theoretical literature, we describe in [Section 2](#) the setting of our model. Our objective is to address three questions pertaining to the voluntary disclosure setting with the possibility of an exogenous signal. First, how does the introduction of an exogenous signal affect the equilibrium disclosure strategy of the manager? This analysis is presented in [Section 3](#). Second, since the presence of an exogenous signal affects the manager's disclosure strategy, how does a change in the probability of an exogenous signal, e.g., through a change in analyst coverage, affect price efficiency? We answer this in [Section 4](#). Finally, how does a change in analyst coverage affect the liquidity of the firm's stock? To answer that, in [Section 5](#) we introduce, and analyze, an extended model

³ [Kelly and Ljungqvist \(2012\)](#) use several measures of liquidity, including the bid-ask spread, which is the measure we use in [Section 5](#). [Ellul and Panayides \(2018\)](#) use a measure of price efficiency that follows [Hasbrouck \(1993\)](#), and is close in nature to our theoretical measure in [Section 4](#). Their measure of liquidity is also the bid-ask spread.

that includes a stylized trading stage à la [Glosten and Milgrom \(1985\)](#). [Section 6](#) presents two possible extensions of the model: [Section 6.1](#) discusses a model where the manager suffers a cost if she does not disclose and the analyst publishes a report, and [Section 6.2](#) discusses a framework where the analyst observes information that is noisier than the information of the manager. [Section 7](#) briefly concludes.

1.1. Related theoretical literature

Our study of voluntary disclosure in the presence of potentially informed traders contributes to two streams of the theoretical literature. The first is the voluntary disclosure literature. To the best of our knowledge, only a few theoretical papers study voluntary disclosure in the presence of a potentially informed market/receiver. In an early contribution, [Dutta and Trueman \(2002\)](#) study a setting in which a firm's manager can credibly disclose private information, but do not know whether the market will react positively or negatively to this information. [Langberg and Sivaramakrishnan \(2008, 2010\)](#) offer two models of voluntary disclosure with an additional analyst, where the analyst's information is orthogonal to the information of the firm; in our model, analysts and the firm potentially learn the same information. This makes the analysis very different. [Einhorn \(2018\)](#) also explores the effect of additional information sources on voluntary disclosure. In her model, in contrast to the present paper, there is a difference between the information that the manager learns and the information that she can disclose, and this difference determines the manager's disclosure strategy. The closest work with an informed receiver is [Ispano \(2016\)](#), whose model, while very different, can be seen as a simplified version of our model with three possible firm values and a specific analyst technology. In this simple version, he proves that the utility of the receiver—which is equivalent to price efficiency in our setting—is increasing with the probability that the receiver is informed. He does not discuss liquidity. [Ebert et al. \(2019\)](#) analyze, in a similar framework to ours, different types of information leaks. See also [Quigley and Walther \(2018\)](#) and [Banerjee and Kim \(2017\)](#) for related, yet different, models of voluntary disclosure where the market may observe additional information.

The second stream of literature studies how changes in one source of information affect the incentives of other parties to acquire and disseminate information. Several papers have considered the endogenous acquisition of private information by investors in a setting where public information is available (see, e.g., [Verrecchia, 1982](#); [Diamond, 1985](#); [Demski and Feltham, 1994](#); [Kim and Verrecchia, 1994](#); [McNichols and Trueman, 1994](#)). A key result in this literature is that better public information crowds out incentives to acquire private information.⁴ [Goldstein and Yang \(2017\)](#) study, in a noisy-REE setting ([Grossman and Stiglitz, 1980](#)), the effect of such crowding out on the over-

all amount of public information. They show that when the crowding out of private information acquisition is taken into account, the overall effect of increased public information is ambiguous and depends on the parameter values and the particular measure of market quality. [Fischer and Stocken \(2010\)](#) study a model in which analysts acquire costly information and then communicate with investors using cheap-talk messages (as in [Crawford and Sobel, 1982](#)). They also find that better exogenous information may crowd out the incentives of analysts to acquire information, and decrease the overall amount of publicly available information.

Our paper complements this literature by analyzing exogenous information and an endogenous, voluntary disclosure decision. In our model, better exogenous information, e.g., due to greater analyst coverage, may crowd out or crowd in voluntary disclosure. Our voluntary disclosure setup is unique, because, although the analyst and the firm observe the same type of information, the information that they reveal publicly is different. The firm discloses only high signals and withholds low signals. The analyst, however, reveals all the information he obtains. In contrast to the papers described above, in our setup the two sources of information *are not perfect substitutes*, that is, an increase in analyst coverage cannot be fully offset by lower corporate voluntary disclosure. The difference between voluntary disclosure and other means of communication leads to different qualitative results: in contrast to [Fischer and Stocken \(2010\)](#) and [Goldstein and Yang \(2017\)](#), even when exogenous information crowds out voluntary disclosure, better exogenous information always has a positive effect on the overall amount of public information. We explain our result in detail in [Section 4](#).

2. Setting

Our model builds on the voluntary disclosure literature initiated by [Grossman \(1981\)](#), [Milgrom \(1981\)](#), and [Dye \(1985\)](#). We consider a firm that is involved in a project, e.g., drug development or oil exploration, which will either succeed or fail. We denote the terminal value of the firm by $x \in \{0, 1\}$ where $x = 1$ following success and $x = 0$ following failure. The ex ante probability of success is $\mu_0 \equiv \Pr(x = 1)$ and the probability of failure is $1 - \mu_0 \equiv \Pr(x = 0)$.

Information structure

With probability $q \in (0, 1)$, the manager of the firm observes additional information about the possible outcome of the project, in the form of a signal s . With probability $1 - q$ the manager does not observe a signal. Information endowment is independent of the realization of s , and therefore the ex ante expected value of s (or x) conditional on an information event also equals μ_0 . The signal may represent, for example, the results of a clinical trial or an oil exploration, information about competing projects/firms or information about relevant macroeconomic conditions. We assume that all players in the game are risk neutral, and thus it is without loss of generality to assume that the signal s is simply the updated probability of success, that is, $\Pr(\tilde{x} = 1|s) = E(\tilde{x}|s) = s$. Hence, we as-

⁴ Another related paper is by [Gao and Liang \(2013\)](#), who study how a firm's commitment to disclosure affects investors' incentives to acquire information. Their focus is on the feedback effect, whereby the firm's manager learns from prices.

sume that $\tilde{s} \in [0, 1]$, with a PDF $f(s)$, a continuous CDF $F(s)$, and $E[\tilde{s}] = \mu_0$.

Remark 1 (Alternative State Space). Most of our results extend to arbitrary continuous distributions of \tilde{x} and \tilde{s} (with bounded or unbounded support). This includes all the results in Sections 3 and 4. The binary structure is only used to simplify the trading stage in Section 5.

Disclosure and pricing

If the manager observes the private signal s , she can voluntarily disclose it to the market. Disclosure is assumed to be costless and credible (verifiable at no cost). As standard in the voluntary disclosure literature, if the manager does not obtain the private signal, she cannot credibly convey that she is not informed. The manager seeks to maximize the market value, or price, of the firm.⁵ For now, assume that risk neutral investors set the market price, P , equal to the expected value conditional on all the available public information, I . That is, $P = E[\tilde{s} | I] = E[\tilde{x} | I]$. Later, in Section 5, we introduce a trading stage that follows Glosten and Milgrom (1985), where prices are set by a centralized market maker.

The setting introduced so far is similar to a standard voluntary disclosure setting with uncertainty about information endowment, which has been studied extensively. The main innovation of our setting is the possibility that the signal s will be made public by an external third party.

Analyst (exogenous signal)

We use financial analysts as our main motivating example, however, any mechanism that induces stochastic public supply of the firm's information, such as news media, competitors, suppliers, social media, regulators etc., will have a similar effect in our model.

To study the interaction between a firm's voluntary disclosure and the potentially informed market, we add to the above setting a financial analyst, who may also learn the realization of the updated probability of success, s . We abstract from strategic considerations of the analyst, and assume that whenever analysts discover information they publish it truthfully.⁶ In the baseline model we assume that, if informed, both the analyst and the manager observe the same information. In Section 6.2 we discuss the case where the analyst's information is less precise than that of the manager.

The likelihood of the analyst to discover information may depend on whether the manager is informed or not. For example, if the information s is the result of a clinical drug trial, it is unlikely that the analyst will discover this information before the manager does. However, if the signal s is information about market conditions, the analyst

may discover this information even when the manager is uninformed. To allow for both types of information, we assume a relatively non-restrictive analyst's information production technology. In particular, assume that the analyst's information production technology is reflected by a pair of conditional probabilities $(g_I(r), g_U(r))$, where $g_I(r) \in [0, 1]$ and $g_U(r) \in [0, 1]$ are the probabilities that the analyst discovers s conditional on the manager being informed and uninformed, respectively. We introduce the parameter r to capture the overall quality and/or quantity of analysts that cover the firm. We refer to r as "analyst coverage." An increase in analyst coverage weakly increases the probability that the analyst becomes informed when the manager is informed and when the manager is uninformed. For simplicity, we assume that g_I and g_U are differentiable, and thus assume $g'_U(r) \geq 0$ and $g'_I(r) \geq 0$, with at least one strict inequality. Note that the ex ante probability that the analyst issues a report is $q \cdot g_I(r) + (1 - q) \cdot g_U(r)$.

Timeline

To summarize our disclosure game, the timeline is as follows.

1. With probability q the manager privately learns the signal s .
2. If the manager is informed, she decides whether to publicly disclose s or not.
3. Analysts learn the signal s with probabilities $g_I(r)$ or $g_U(r)$, depending on the outcome of stage 1. An informed analyst immediately discloses s to the market.
4. Following the disclosure or lack of disclosure by both the manager and the analyst, market participants update their beliefs about the expected value of the firm/project.
5. The price of the firm is determined, and the manager is compensated accordingly. We first assume risk neutral pricing, and in Section 5 we specify a market mechanism that generates the price.

The setting and all the parameters of the model are common knowledge.

Remark 2 (Alternative Timing). The information that the manager and the analyst may learn and disclose is identical. Thus, the manager's disclosure is relevant only in the case the analyst has not published a report. This implies that even if the manager knows whether the analyst has published, or about to publish, a report before making her disclosure decision (that is, even if stage 3 is before stage 2), the equilibrium is essentially the same: following a disclosure by the analyst the manager is indifferent whether to disclose or not, and following no analyst report the manager's strategy is identical to her strategy in the current model.

3. Analysis of the disclosure decision

3.1. Equilibrium disclosure strategy

Given the realized signal, an informed manager chooses a disclosure strategy that maximizes the expected firm price. If s is publicly disclosed either by the manager or by

⁵ As standard in the literature, we take the performance-based compensation of the manager as given. Such compensation may be an optimal contract when the manager has additional activities, which are left unmodeled, that demand effort (as in Holmström, 1979; Holmström, 1999). Such compensation is also optimal when the market / receiver wishes to price the firm "correctly" (Hart et al., 2017).

⁶ It is immediately obvious that all of our results are robust to an analyst's reporting strategy that is potentially biased, as long as the analyst always issues a report when obtaining information and the analyst's forecast follows a separating strategy. For an example and additional references see Beyers and Guttman (2011).

the analyst—an event we denote by “D”—the price of the firm equals its expected value, i.e.,

$$P^D(s) \equiv E[\tilde{x}|s] = s.$$

Denote by “ND” the event that neither the manager nor the analyst disclosed s , and by P^{ND} the price following such an event. P^{ND} is the market’s belief about the firm’s expected value following no disclosure., i.e., $P^{ND} \equiv E[\tilde{x}|ND]$.

The manager’s disclosure decision affects the price only when s is not disclosed by the analyst. Thus, though an informed manager does not know whether the analyst will be informed or not, she conditions her decision only on the event that the analyst will not be informed. When the analyst is not informed, an informed manager’s optimal strategy is to disclose s if and only if $P^D(s) > P^{ND}$. While $P^D(s)$ is increasing in s , P^{ND} is independent of the manager’s type. Therefore, any equilibrium disclosure strategy is characterized by a threshold signal—which we denote by σ —such that an informed manager discloses her signal if and only if $s \geq \sigma$.

The price following no disclosure by the manager or the analyst, P^{ND} , depends on the market’s belief about the manager’s disclosure strategy. If the market believes the manager uses a disclosure threshold σ , then the price following no disclosure is given by

$$P^{ND}(\sigma) \equiv E[\tilde{x}|ND, \sigma] = \frac{(1 - q) \cdot (1 - g_U(r))E[\tilde{s}] + qF(\sigma) \cdot (1 - g_I(r)) \cdot E[\tilde{s}|s < \sigma]}{(1 - q)(1 - g_U(r)) + qF(\sigma)(1 - g_I(r))}. \tag{1}$$

The price is a weighted average of the prior mean and the mean conditional on withholding signals below σ , with weights representing the conditional probabilities that the manager is informed and uninformed, given that no analyst report was published. Thus, for any exogenously given disclosure threshold $\sigma \in (0, 1)$ the price given no disclosure is lower than the prior mean, that is, $P^{ND}(\sigma) < E[\tilde{s}] = \mu_0$.

Our disclosure model generalizes Dye (1985) and Jung and Kwon (1988) to a setting that contains an additional stochastic public revelation mechanism. Formally, those models are a particular case of our setting in which $g_I(r) = g_U(r) = 0$. It is easy to extend the analysis in Jung and Kwon (1988) to our setting and show that a threshold equilibrium exists, and that it is unique.

Fact 1. There exists a unique equilibrium to the disclosure game, in which an informed manager discloses if and only if the signal s is greater than a disclosure threshold σ^* . σ^* is the signal that makes the manager indifferent between disclosing or withholding. The disclosure threshold is given by the unique solution of the condition

$$\sigma^* = P^{ND}(\sigma^*). \tag{2}$$

Similarly, it is easy to show that the derivative $\frac{\partial P^{ND}(\sigma)}{\partial \sigma}$ is less than one in the neighborhood of σ^* , and thus a change in any parameter that increases or decreases the function $P^{ND}(\sigma)$ for any threshold σ , also increases or decreases the equilibrium threshold σ^* .⁷ If, for example, a

change in r increases the price following no disclosure for any exogenously given disclosure threshold, then it must also increase the equilibrium threshold (that is, decrease disclosure). This is formalized in the following fact.

Fact 2. The equilibrium disclosure threshold σ^* is increasing (decreasing) in r , if and only if $P^{ND}(\sigma)$ is increasing (decreasing) in r .

3.2. Effect of analyst coverage on disclosure strategy

In this section we analyze the main comparative static of the disclosure game—how the level of analyst coverage, r , affects the manager’s equilibrium disclosure threshold, σ^* .

Based on Fact 2, to study the effect of analyst coverage on corporate disclosure, we can study how analyst coverage affects the price given no disclosure for an exogenous disclosure threshold σ , i.e., $\frac{\partial P^{ND}(\sigma)}{\partial r}$. Note from (1) that, for any exogenous disclosure threshold σ , $\frac{\partial P^{ND}(\sigma)}{\partial g_I(r)} > 0$ and $\frac{\partial P^{ND}(\sigma)}{\partial g_U(r)} < 0$. Greater $g_I(r)$ means that the analyst is more likely to discover and publish s when the manager is informed. Thus, no disclosure when $g_I(r)$ is greater implies that it is less likely that the manager is informed and withholds negative information. Therefore, an increase in $g_I(r)$ increases P^{ND} . In contrast, greater $g_U(r)$ means that the analyst is more likely to discover and disclose s when the manager is uninformed. Thus, no disclosure when $g_U(r)$ is greater implies that it is more likely that the manager is informed and withholds negative information. Therefore, an increase in $g_U(r)$ decreases P^{ND} . The overall effect of an increase in r on the price given no disclosure is

$$\frac{\partial P^{ND}(\sigma)}{\partial r} = \frac{\partial P^{ND}(\sigma)}{\partial g_I(r)} g'_I(r) + \frac{\partial P^{ND}(\sigma)}{\partial g_U(r)} g'_U(r).$$

Since both $g_I(r)$ and $g_U(r)$ increase in r , the overall effect of changes in r on P^{ND} is not clear. Without further assumptions about the functions $g_I(r)$ and $g_U(r)$, one cannot conclude whether an increase in analyst coverage increases or decreases the equilibrium disclosure threshold. Next, we provide the condition that determines the effect of a change in r on the disclosure strategy, and thus on corporate disclosure.

3.2.1. Informed analyst ratio and effect on disclosure

In order to study the effect of analyst coverage on the equilibrium disclosure strategy, it is useful to consider the following function

$$m(r) \equiv \frac{\Pr(\text{analyst is uninformed} \mid \text{manager is uninformed})}{\Pr(\text{analyst is uninformed} \mid \text{manager is informed})} = \frac{1 - g_U(r)}{1 - g_I(r)}. \tag{3}$$

$m(r) \in [0, \infty)$ is the ratio between the likelihood that the analyst does not discover and discloses s when the manager is uninformed and the likelihood that the analyst does not disclose s when the manager is informed. For convenience, we henceforth refer to $m(r)$ as the “informed analyst ratio.”

⁷ In fact, $\frac{\partial P^{ND}(\sigma)}{\partial \sigma} |_{\sigma=\sigma^*} = 0$. See Fact 3 in the Appendix.

Denote by σ_D^* the disclosure threshold in a model with no analyst, i.e., where $g_U = g_I = 0$. This is the classic [Dye \(1985\)](#) model. We first show that the size of $m(r)$ determines whether the presence of an analyst increases or decreases voluntary disclosure compared to the [Dye \(1985\)](#) model.

Lemma 1. The firm discloses less information compared to the case where an analyst is not available if and only if the informed analyst ratio is greater than one; that is

$$\sigma^*(r) > \sigma_D^* \iff m(r) > 1.$$

Proof. Using (1), $p^{ND}(\sigma, r)$ can be rewritten as

$$p^{ND}(\sigma, r) = \frac{(1-q)E[\tilde{x}] + q \cdot m(r)^{-1} \cdot F(\sigma)E[\tilde{x} | s < \sigma]}{1 - q + q \cdot m(r)^{-1} \cdot F(\sigma)}. \quad (4)$$

By (4) and the fact that $E[\tilde{s}] > E[\tilde{s} | s < \sigma]$, it is clear that $p^{ND}(\sigma, r)$ is increasing in $m(r)$. By (3), $m = 1$ when $g_I = g_U = 0$. Thus, $p^{ND}(\sigma, r) > p^{ND}(\sigma, r)|_{g_I=g_U=0}$ if and only if (iff) $m(r) > 1$. The lemma then follows from [Fact 2](#). \square

We now turn to the effect of changes in analyst coverage on the level of voluntary disclosure, i.e., on the disclosure threshold. The following proposition shows that this effect depends on the directional change in $m(r)$ as r changes.

Proposition 1. In equilibrium, analyst coverage crowds out corporate voluntary disclosure if and only if $m'(r) > 0$, that is,

$$\frac{\partial \sigma^*}{\partial r} > 0 \iff m'(r) > 0.$$

Proof. By (4) and the fact that $E[\tilde{s}] > E[\tilde{s} | s < \sigma]$, it is clear that $p^{ND}(\sigma, r)$ is increasing in $m(r)$. Thus, $\frac{\partial p^{ND}(\sigma)}{\partial r} > 0$ iff $m'(r) > 0$. The lemma then follows from [Fact 2](#). \square

Greater $m(r)$ means that the analyst is *relatively* more likely to be uninformed when the manager is uninformed than when the manager is informed. Thus, if the analyst does not report, this signals that the manager is more likely to be uninformed. Formally, as shown by (4),

$$\begin{aligned} & \Pr(\text{manager is uninformed} | \text{ND}) \\ &= \frac{1-q}{1-q+q \cdot m(r)^{-1} \cdot F(\sigma)}. \end{aligned}$$

Therefore, higher $m(r)$ gives the manager a higher payoff in the case that the analyst does not publish a report, and thus a higher incentive to withhold. Note that, as discussed above, the probability that the analyst becomes informed does not enter the manager's payoff function in any way except through p^{ND} .

3.2.2. Does analyst coverage crowd out or crowd in voluntary disclosure?

[Proposition 1](#) shows that the effect of analyst coverage on voluntary disclosure depends on properties of the analyst's information production. If an increase in analyst coverage is more likely to uncover information that the manager is unaware of, then [Proposition 1](#) predicts that voluntary disclosure will increase with analyst coverage

(crowding in). If, however, an increase in analyst coverage is more likely to uncover information that is known to the manager, then voluntary disclosure is expected to decrease (crowding out).

Whether the analyst is more likely to possess information when the manager is informed or when the manager is uninformed depends on the nature of the information. For some types of information, such as internal accounting information, clinical trials, and drilling results, it is reasonable to assume that if the manager does not observe the information it is very unlikely that the analyst will learn it. For such information, crowding out should prevail. For other types of information, such as information about market conditions, competitors, and new regulations, it is reasonable that the analyst will learn the information even if the manager does not. For such information, both crowding in and crowding out are theoretically plausible. Our model does not restrict the nature of the information and accommodates both crowding out and crowding in.

The empirical evidence that is reviewed in the Introduction suggests that, on average, analyst coverage crowds out voluntary disclosure. [Anantharaman and Zhang \(2011\)](#) as well as [Balakrishnan et al. \(2014\)](#) report that an exogenous decrease in analyst coverage led firms to provide more earning guidance (which is the most commonly used measure of voluntary disclosure). This empirical evidence, viewed through the lens of our model, suggests that analysts mostly uncover information that is already available to the manager. In technical terms, it suggests that information structures with $m'(r) > 0$ are more prevalent.

Our model can also be used to guide future empirical research in order to gain insight on the information sources of analysts. For example, future research may analyze the effect of analyst coverage on voluntary disclosure at the industry level (or some other partition of the data). Our model can then be used to suggest whether analysts who cover a specific industry are more likely to learn information when the manager is informed or uninformed.

We offer below two relatively simple examples of such information structures, which we find appealing and realistic. In both examples $m'(r) > 0$.

Example 1 (Private Inquiry and Leaks). Suppose that the manager learns \tilde{s} with probability q . The analyst has two potential sources of information, one within the firm and the other external. Examples for external sources could be information about the industry or macroeconomic conditions. Further assume that the probability that the analyst learns s from an external source is r and this probability is independent of whether the manager is informed or not. One interesting case of this example is $r = 0$, which may represent the results of a clinical trial or oil and gas drilling, that are unlikely to be available to the analyst and not to the manager.

The inside source of information captures information that is "leaked" to the analyst from within the firm.⁸ Such information can be observed by the analyst only when the

⁸ [Green et al. \(2014\)](#) show that access to management remains an important source of information for analysts even following Regulation Fair Disclosure (Reg FD).

manager is informed. Suppose that the probability that the analyst learns s from insiders, conditional on the manager being informed, is $\delta(r) \in (0, 1)$. Naturally we assume that an increase in analyst coverage increases the probability of leaks. For simplicity, we assume that $\delta(r)$ is differentiable, and $\delta'(r) > 0$. In this example, we obtain $g_U(r) = r$ and $g_I(r) = r + (1 - r)\delta(r)$, so that $m(r) = \frac{1}{1 - \delta(r)}$.

Example 2 (Conditionally Independent Information Endowment). Suppose that with probability $\omega \in (0, 1)$ some information event occurs and with probability $1 - \omega$ no information event occurs. If no information event occurs, the firm's expected value remains the prior mean (μ_0). However, if an information event occurs, it generates a new probability of success s , which equals to the updated expected value of the firm.

Conditional on an information event occurring, the probability that the analyst discovers s is r , and the probability of the manager discovering s is $\frac{q}{\omega}$ (so the overall probability that the manager discovers s is q). Assume that the information endowment events of the manager and the analyst are independent, conditional on an information event. This structure implies

$$g_U(r) = \frac{\omega(1 - \frac{q}{\omega})r}{1 - q} = \frac{\omega - q}{1 - q} \cdot r \text{ and } g_I(r) = \frac{\omega \frac{q}{\omega} r}{q} = r,$$

so that $m(r) = 1 + \frac{1 - \omega}{1 - q} \cdot \frac{r}{1 - r}$

One can easily verify that $m'(r) > 0$ in both examples. Thus, by Proposition 1, the manager's disclosure threshold increases in analyst coverage ($\frac{\partial \sigma^*}{\partial r} > 0$). In other words, in both of these examples an increase in analyst coverage crowds out voluntary disclosure.

3.3. Analyst coverage and public information

An increase in analyst coverage, r , by definition increases the probability that the signal will be discovered and disclosed by analysts, and thus has a direct effect of improving the available public information. Such an increase, however, also affects the firm's voluntary disclosure.

In the case of a crowding-in effect, the firm also provides more information under increased analyst coverage, and thus the overall effect on public information is clearly positive. One can formally prove that the quality of public information, as measured using Blackwell's informativeness criterion, improves.⁹ Therefore, it is clear that in the crowding-in case the price efficiency and liquidity, as de-

⁹ Formally, in our setup a "public information structure," or "experiment," can be defined as a function $\psi: [0, 1] \rightarrow [0, 1]$, where $\psi(s)$ is the probability that a signal s is disclosed publicly, while with probability $1 - \psi(s)$ the signal is not disclosed by the analyst or the firm (ND). Now consider two levels of analyst coverage, r and r' , where $r' > r$, and the respective public information structures, ψ and ψ' . In the case of a crowding-in effect ($m'(r) < 0$ in Proposition 1), $\psi'(s) > \psi(s)$ for any s . Thus, ψ can be represented as a garbling of ψ' using a garbling function λ : for any s , there exists $\lambda(s) \in (0, 1)$ such that $\psi(s) = \lambda(s)\psi'(s)$. The result that ψ' is more Blackwell-informative than ψ follows immediately from this representation and Blackwell's informativeness theorem (Blackwell, 1953).

finied in the following sections, increase in analyst coverage.

However, in the crowding-out case ($m'(r) > 0$ in Proposition 1), which is supported by the empirical evidence, the Blackwell informativeness criterion does not hold. This is because greater coverage increases the probability that the value of some types will be disclosed, but decreases this probability for other types (types between the previous and the new disclosure thresholds, that cease to be disclosed by the manager). In the next two sections, we study the effect of analyst coverage on two common measures of quality of overall public information. The only case that is left to study is the case in which $m'(r) > 0$, i.e., *analyst coverage crowds out voluntary disclosure*.

4. Price efficiency

In this section we assess the overall effect of an increase in analyst coverage. As discussed in the previous section, we analyze the more ambiguous case, which is also supported by the empirical evidence, in which analyst coverage crowds out voluntary disclosure. This effect can be decomposed into two parts:

- A change in the probability that the signal s is made public, either by the manager and/or by the analyst. The probability of this event is given by

$$q \cdot g_I(r) + q(1 - g_I(r))(1 - F(\sigma^*)) + (1 - q)g_U(r).$$

When the manager's equilibrium disclosure threshold, σ^* , is increasing in analyst coverage r , it is not clear whether this probability increases or decreases following an increase in r .

- Market uncertainty regarding s in case it does not become public. An increase in r affects the distribution of types given no disclosure, and hence the uncertainty given no disclosure.

Any measure of information must take into account both parts (and thus, for example, the probability of disclosure does not capture the quality of overall information available to the market). In the next section we suggest a measure of price efficiency, which is the inverse of the expected squared distance of prices from the fundamental value. We then show that an increase in analyst coverage always increases price efficiency according to this measure.

4.1. A measure of price efficiency

In our model, when information is made public either by the manager or by the analyst, the price perfectly reflects all the information, i.e., the price is $P^D = E[\tilde{x}|s] = s$. When information is not made public the price is on average correct, but it is a noisy measure of the signal (that the manager may either not know or is actively withholding), $P^{ND} = E[\tilde{s} | ND]$.

To measure how efficiently prices reflect information about future cash flows, we adopt the commonly used expected squared deviation between the market price and the signal s . Our price efficiency measure, which we refer to as PEF, is given by

$$PEF \equiv -E \left[(\tilde{s} - \tilde{P})^2 \right]. \tag{5}$$

PEF may represent the “social” benefit from having a price that is close to the fundamental, or the externalities and gains that are obtained from the informativeness of prices. Note that this measure is in line with our assumption of risk neutral pricing: a social planner who wishes to maximize efficiency will choose $P = E[\tilde{s} | I]$, where I is all the available information.

Another interpretation of PEF is that it is the variance of the noise in the price relative to the true underlying value s . Thus, higher price efficiency means a decrease in the residual uncertainty of prices (the future movement of prices when the real cash flows x will be realized or revealed).

4.2. Analyst coverage and price efficiency

We have discussed above the challenge in determining even the directional effect of changes in analyst coverage, r , on price efficiency. One of our main results is that an increase in analyst coverage always increases price efficiency.

Proposition 2. Price efficiency increases in analyst coverage, i.e.,

$$\frac{dPEF(r)}{dr} > 0.$$

The formal proof of the proposition is quite involved, and hence is relegated to the Appendix. The economic intuition relies on the qualitative difference between the two information sources: the analyst and the firm. First, following an increase in coverage, the analyst discloses the firm’s value with higher probability. Second, as we pointed out before, under the assumption that $m'(r) > 0$, an increase in coverage leads to less corporate disclosure: managers disclose only if the value is above a higher threshold. We show in the proof that the change in corporate disclosure always has only a second-order effect on public information compared to the increase in analyst disclosure.

The intuition is as follows. The change in corporate disclosure affects only types that disclose under low coverage and withhold under a higher coverage. Because the affected types are close to the disclosure threshold, which equals the price following no disclosure (P^{ND}), their pricing after they cease to be disclosed by the firm is close to the pricing they obtain following disclosure. Thus, the mispricing that arises because those types cease to be disclosed by the manager is minimal (second order).

The results of Proposition 2 are supported by Ellul and Panayides (2018), who measure price efficiency using the methodology of Hasbrouck (1993). This methodology uses vector autoregression to statistically estimate the difference between trading prices and the stock’s estimated fundamental price, and measure price inefficiency as the standard deviation of this difference. Our measure of PEF is evidently the same, though it is developed within a much simpler, static model. Ellul and Panayides (2018) find that price efficiency decreases following the termination of analyst coverage, and that the decrease is more moderate for firms that increased the number of news releases in the post-termination period, and for firms that issue earning guidance. These results are in line with the predictions of this section.

5. Informed trading and liquidity

The results in the previous section examine the effect of analyst coverage on a theoretical measure of price efficiency. While price efficiency is a very appealing theoretical construct, empirically measuring or estimating it is not easy. In this section, we study the effect of analyst coverage on liquidity, which is a measure of information asymmetry that is common in the empirical literature and can be measured directly. Our measure of liquidity is the bid-ask spread, which is relatively easy to estimate. We analyze how the expected bid-ask spread, which reflects the information asymmetry that remains after the disclosure game, is affected by analyst coverage. Note that the bid-ask spread in our model reflects difference in information quality among market participants, while the price efficiency measure analyzed in the previous section reflects the uncertainty of the market overall about the fundamentals. Although these two measures are related, the two constructs capture different aspects of the information environment.

We extend our disclosure model by adding a stylized trading stage. Trading occurs after the manager’s potential voluntary disclosure decision and after the potential release of the analyst’s report. Let I be the public information by the end of the disclosure stage, then $\mu \equiv \Pr(x = 1 | I)$ is the public belief about the firm’s terminal value at the beginning of the trading stage.

The trading stage is a static version of the Glosten and Milgrom (1985) model (henceforth GM). The trading stage involves a competitive market maker and a single trader. The trader can either buy or sell one unit (share) of the firm’s stock. With probability $1 - p$ the trader is a “liquidity trader,” who sells or buys independently of the firm’s value (for example, due to a liquidity shock). The liquidity trader chooses to sell or to buy one unit with equal probabilities (our results are robust to changes in the probabilities). With probability $p \in (0, 1)$ the trader is *strategic* and trades to maximize his trading profit given his information (we assume this trader obtains a payoff of zero in case he does not trade). With an exogenous probability $\chi \in (0, 1]$ the strategic trader is *informed*, and knows the firm’s terminal value, x . Thus, the trader is strategic and informed with probability $p\chi$, and with probability $p(1 - \chi)$ the trader is strategic but does not have additional information.

The risk neutral market maker does not have private information about the firm value or the type of the trader. The market maker operates in a competitive market (which is not modeled), and sets prices that lead to zero expected profit. Given the initial belief μ , the bid price, $b(\mu)$, is set to equal the expected value of the asset conditional on the trader selling a share. Similarly, the ask price, $a(\mu)$, is set to equal the expected value of the asset conditional on the trader buying a share. The bid-ask spread, denoted by $\Psi(\mu)$, is the difference between the ask and the bid prices above, that is,

$$\Psi(\mu) \equiv a(\mu) - b(\mu). \quad (6)$$

We now provide several technical facts about the extended game. For brevity, we describe them informally; the formal treatment and proofs appear in A.3. As is standard in

GM, the strategic trader buys if $x = 1$ and sells if $x = 0$. Thus, a purchase by the trader is a positive signal about the value of the asset, and a sale by the trader is a negative signal about this value. As a result, $b(\mu) < \mu < a(\mu)$ for any $\mu \in (0, 1)$ and $p \in (0, 1)$, and the spread is always positive. The spread is larger when there is higher uncertainty about the value of x , which happens in intermediate levels of μ ; when $\mu = 0$ or $\mu = 1$ the value is known before trading and the spread is zero. An important property, which is used in our proofs, is that the spread is not only non-monotone, but is also concave (that is, inverse U-shaped).

In the Appendix we also analyze the manager's disclosure decision in the extended model. We show that the disclosure policy is as in the basic model, that is, according to the threshold σ^* that is defined in [Fact 1](#). The proof relies on the fact that the ex ante price is equal to μ , and thus that an informed manager of type σ^* expects a price of σ^* .

5.1. Disclosure and liquidity

The initial public belief μ in the trading stage is a result of the information that is disclosed or not disclosed by the manager and the analyst at the disclosure stage. In this section we study how the parameters of the disclosure game affect the liquidity in the trading stage.

Our measure of illiquidity, $IL(q, r) > 0$, which depends on the parameters of the disclosure game, q and r (as well as the parameters of the trading stage, p and χ , which are treated as given), is the expected bid-ask spread, and is given by

$$IL(q, r) \equiv E[\Psi(\mu) | q, r].$$

When we refer to liquidity we refer to $IL(q, r)^{-1}$.

We are interested in the effect of analyst coverage, r , on liquidity. The difficulty in the analysis is similar to the one described in [Section 4](#), and stems from the fact that an increase in r has an ambiguous effect on the probability that the signal becomes public, as well as the effect of the underlying uncertainty following no disclosure. IL, however, captures a different economic construct than PEF. In particular, expected liquidity is not a linear function of PEF, and hence [Proposition 2](#) does not imply that the expected liquidity increases in r . For example, if a certain signal s is disclosed with higher probability following an increase in r , then this clearly has a positive effect on price efficiency because disclosure results in $P = s$. However, since, as we have explained above, the spread is non-monotone (see also [Lemma 3](#) in the Appendix), disclosure of s may actually decrease liquidity if $\Psi(\sigma^*) < \Psi(s)$. Thus, the direct effect of an increase in coverage on IL is more nuanced than the effect on PEF. Nevertheless, it is possible to show that analyst coverage always has a total positive effect on liquidity:

Proposition 3. The expected bid-ask spread, $IL(q, r)$, is decreasing in r for any $q \in (0, 1)$, that is

$$\frac{dIL(q, r)}{dr} < 0.$$

The proof of [Proposition 3](#) is in the Appendix. The proof uses similar intuition as in the proof of [Proposition 2](#) to

show that the change in disclosure threshold plays a second-order effect where the direct effect is of first order. This is again because the change in the disclosure threshold affects mostly firms that are close to the threshold and therefore are priced relatively accurately even when their manager chooses not to disclose.

The result of [Proposition 3](#), which provides additional motivation for the informational benefit of analyst coverage, is consistent with the empirical findings of the papers we have presented in the introduction. [Kelly and Ljungqvist \(2012\)](#) and [Ellul and Panayides \(2018\)](#) find that following an exogenous negative shock to analyst coverage, there is a decrease in the liquidity of the affected firms. [Balakrishnan et al. \(2014\)](#) as well as [Ellul and Panayides \(2018\)](#) find evidence that the decrease in liquidity is partially reversed by an increase in voluntary disclosure (in the form of earning guidance and press releases), but overall liquidity still decreases, in line with the results of this section.

6. Extensions

In this section we discuss two extensions of the model. First, we analyze a model where the manager may suffer a cost if the analyst publishes a report following no disclosure. Next, we discuss how changes in the information structure may affect our main results.

6.1. Disciplinary role of analyst coverage

In our main model, if the analyst publishes a report, the manager's payoff is the same regardless of whether she has disclosed or not. It seems natural to believe that, conditional on the signal being revealed anyway, the manager would prefer to disclose that information by herself. If the analyst issues a report and the firm does not disclose, the firm could be accused of violating its "duty to disclose" by hiding unfavorable information. Such an event could trigger costly litigation, and such a possibility incentivizes the manager to precautionarily disclose information.¹⁰ This precautionary motive depends on the probability that an analyst report will be published, and thus on analyst coverage. In what follows we introduce precautionary disclosure into our main model.

6.1.1. A model with litigation costs

We extend the model in [Section 2](#) to include a cost that the manager bears if an analyst report is published following no corporate disclosure. We refer to this event as "analyst only" and denote it by AO, and we refer to the cost as the "litigation cost," although it may also represent other costs, such as reputation cost, etc. To capture the idea that the manager is penalized for actively withholding information, we let this cost increase with the probability that the

¹⁰ See [Marinovic and Varas \(2016\)](#) and [Dye \(2017\)](#) for an additional discussion on litigation risk due to nondisclosure. [Marinovic and Varas \(2016\)](#) study such litigation risk in a dynamic costly disclosure setup, which is very different from ours. [Dye \(2017\)](#) also extends [Dye \(1985\)](#) by incorporating litigation risk; however, he presents specific litigation cost that does not affect the equilibrium.

manager has private information conditional on observing only an analyst report. Formally, we denote this cost by $C(q^{AO})$, where q^{AO} is the posterior belief of the market that the manager is informed following an “only report” event, and we assume that $C(0) = 0$ and $C' > 0$.¹¹

When an informed manager who has observed a signal s makes a disclosure decision, she compares her payoff from disclosure, $P^D(s) = s$, to her expected payoff from no disclosure, which is now

$$(1 - g_l)P^{ND} + g_l(s - C(q^{AO})).$$

Note that the payoff from no disclosure is less sensitive to s compared to the payoff from disclosure. Thus, as in the basic model, the manager discloses if and only if s is greater than a disclosure threshold, which we denote by σ_C^* . For a given cost function C , σ_C^* satisfies the indifference condition

$$\sigma_C^* = P^{ND}(\sigma_C^*) - \frac{g_l}{1 - g_l} C(q^{AO}(\sigma_C^*)), \tag{7}$$

where $P^{ND}(\sigma)$ is defined as in (1) and

$$q^{AO}(\sigma) \equiv \frac{qF(\sigma)g_l}{(1 - q)g_u + qF(\sigma)g_l}. \tag{8}$$

The following lemma presents the main properties of the equilibrium.

Lemma 2. The game with litigation costs admits a unique equilibrium. In this equilibrium the manager discloses iff $s > \sigma_C^$, where σ_C^* is defined as in (7). The equilibrium has the following properties:*

1. Litigation costs together with analyst coverage induce precautionary disclosure: if $g_l > 0$, then $\sigma_C^* < \sigma^*$.
2. Litigation costs cannot induce full disclosure: if $g_l < 1$, then $\sigma_C^* > 0$ for any cost function C .

Part 1 of the lemma asserts that, as expected, in a model with analyst coverage, litigation costs have a disciplinary role and lead to more corporate disclosure compared to the case where such costs are not present. The increased disclosure over the interval $s \in [\sigma_C^*, \sigma^*]$ is due to “precautionary motive.” Higher costs lead to greater precautionary disclosure. This comparative static predicts that large firms, which may be more exposed to litigation risks, would, all else equal, voluntarily disclose more information compared to smaller firms.

Part 2 of the lemma asserts, maybe more surprisingly, that such costs, no matter how high, cannot prevent the manager from withholding some bad news. The reason is that litigation costs decrease as disclosure increases, because greater disclosure means that the manager is less likely to be informed given an “only report” event. This is formalized by the fact that $q^{AO}(\sigma)$ is increasing in σ (Eq. (8)). The posterior $q^{AO}(\sigma)$, and thus the litigation cost, approaches zero as σ approaches zero, and thus the lowest type always has an incentive to withhold news.

¹¹ We continue to assume, as in Section 2, that an informed manager does not know whether the analyst is informed or not. In the basic model this assumption is not restrictive (Remark 2), but here it is: if an informed manager knows that the analyst is informed (for example, because the analyst reports before the manager's disclosure decision) then she can simply disclose in this case and never pay the cost. Such a setting is equivalent to our main model.

6.1.2. Analyst coverage and precautionary disclosure

In this section we discuss how analyst coverage affects precautionary disclosure. We ask whether the term $\sigma^* - \sigma_C^*$ increases or decreases in r . This complements the analysis in Section 3.2 on the effect of r on σ^* , that is, disclosure due to price maximization.

The incentive of the manager to exercise precautionary disclosure depends on the term

$$\frac{g_l}{1 - g_l} C(q^{AO}(\sigma)) \tag{9}$$

(see Eq. (7)); the greater this term is for any given σ , the greater the difference $\sigma^* - \sigma_C^*$ is. An increase in r has two effects on the incentive to precautionary disclosure. First, more coverage increases the probability that the analyst publishes a report and, therefore, also the risk that an informed manager will bear a litigation cost if she does not disclose. This increased risk is represented by the term $\frac{g_l}{1 - g_l}$, which is increasing in g_l and, therefore, also in r .

Second, more coverage affects the size of the litigation cost through a change in q^{AO} , the belief that the manager is informed following no disclosure by the manager and a report by the analyst. This second effect is ambiguous, because q^{AO} may be increasing or decreasing in r . From (8) it can be easily seen that $\frac{\partial q^{AO}}{\partial r} > 0$ if and only if $(\frac{g_l}{g_u})' > 0$. Note that this condition is different from the condition in Proposition 1, which is $m'(r) = (\frac{1 - g_u}{1 - g_l})' > 0$. The intuition, however, is similar to the intuition behind Proposition 1: if the analyst is relatively more likely to uncover information that is already available to the manager, then an analyst report is a stronger signal that the manager is withholding unfavorable information.

The overall effect of analyst coverage on precautionary disclosure therefore depends on $q^{AO}(r)$ and the cost function $C(\cdot)$. If $(\frac{g_l}{g_u})' < 0$ and $\frac{dC}{dq^{AO}}$ is sufficiently high, then an increase in r may even decrease the incentive for precautionary disclosure (the term in (9)). In other examples, an increase in analyst coverage will increase precautionary disclosure. In Example 2 of Section 3.2, the term $\frac{g_l}{g_u}$ is constant, and thus q^{AO} is independent of r . Therefore, only the first effect is present (i.e., the incentive for precautionary disclosure, (9), increases in r due to an increase in g_l), and $\sigma^* - \sigma_C^*$ is increasing in r . Even in this simple example, the effect of r on the total disclosure threshold, σ_C^* , is ambiguous, as it depends also on the change in σ^* .

6.2. Analyst observes a noisier signal

So far we have assumed that the analyst and the firm manager have a potential to learn the same information. A natural extension is the case in which the analyst's information is less precise than the manager's information. Formally, assume a model similar to the one presented in Section 2, where the information endowment of the analyst and the manager is uncertain and possibly correlated, except that now the analyst only observes a noisy signal of s , which we denote by s^a . Following an analyst report that is not accompanied by a manager's disclosure, in contrast to the basic setup, some uncertainty about s remains. This uncertainty is captured by a posterior $f(s|s^a)$.

Note first that in this model, in contrast to the basic model (see Remark 2), the order of moves matters. If the manager discloses before the analyst and does not know the analyst's signal s^a , then she is uncertain regarding her payoff in case she does not disclose. Because the manager's expectations about this payoff depend on her private information, the analysis is convoluted and the model becomes intractable. Hence, we focus here on the case in which the manager discloses after the analyst. In such a case, a threshold equilibrium exists and it is unique. In this equilibrium, the manager discloses according to a threshold that depends on the analyst's report, $\sigma^*(s^a)$, and discloses following no report using a threshold $\sigma^*(\emptyset)$. For brevity, we shall not provide a formal characterization of the model and instead discuss some of its properties.

First, because there are multiple thresholds, it is more difficult to measure how much information is disclosed by the manager, and the effect of a change in analyst coverage cannot necessarily be described as “crowding out” or “crowding in.” To see why, note that in this model an analyst's report informs the market not only of the fundamental value of the firm s , but also of the information endowment of the manager. An increase in coverage (and thus in the probabilities g_I and g_U) may decrease the probability that the manager is informed given that no analyst report is published, thus increasing $\sigma^*(\emptyset)$, and at the same time increase the probability that the manager is informed given an analyst report, thus decreasing $\sigma^*(s^a)$ for any s^a .¹²

Second, our results regarding the effect of analyst coverage on market quality continue to hold given additional assumptions on the analyst's information production technology (that is, g_I , g_U , and s^a). As in the basic model, the direct effect of an increase in analyst coverage on market quality continues to be of first order compared to the effect of the change in corporate disclosure. This can be proven using a similar, albeit more complex, analysis as in Sections 4 and 5. Our results continue to hold as long as we make additional assumptions to assure that public information is sufficiently better following an analyst report compared to no report. A.6 provides a more formal treatment of price efficiency in a model with a noisy analyst signal. Though a full analysis is complex, we show that in the simple case of $g_U = g_I$, that is, when the information endowment of the manager and the analyst are uncorrelated, price efficiency never deteriorates when coverage increases, and strictly increases in a normal distribution example. Though a similar analysis regarding liquidity is more complex, we believe similar results can be obtained.

7. Concluding remarks

The vast theoretical literature on voluntary disclosure has focused on settings with a single information provider. In practice, however, the corporate disclosure environment

is complex and often characterized by several agents who may obtain private information. Financial analysts are one example of such agents. In this paper we have studied how the possibility that the firm's private information may be revealed by a third party (such as an analyst, the media, a regulator, social media, competitors, suppliers, and rating agencies) affects the firm's voluntary disclosure policy and the overall information available to the market. Our model demonstrates that an increase in analyst coverage can either crowd out or crowd in corporate voluntary disclosure—depending on the information structure. The empirical literature is consistent with a crowding-out effect.

When analyst coverage crowds out corporate voluntary disclosure, the effect of an increase in analyst coverage on the quality of overall information is a result of two opposing effects—analysts reveal more information where the firm discloses less information. In order to study the effect of analyst coverage on the quality of information available to the market, we use two common measures of market quality. The first measure is the variance of investors' beliefs (or the future volatility of prices), which has a natural interpretation of price efficiency in our model as it reflects the extent to which current prices reflect the fundamentals. The second measure is the expected bid-ask spread, which measures illiquidity that arises from information asymmetry. In order to calculate the liquidity measure, we introduce a trading stage à la [Glosten and Milgrom \(1985\)](#) that follows the disclosure game. Our model shows that an increase in analyst coverage increases market efficiency and liquidity even when analyst coverage crowds out voluntary disclosure. The relative importance of corporate versus third-party disclosure affects the balance between negative and positive information that is revealed to the market, which in turn determines the quality of public information and other properties such as the skewness of returns. We have demonstrated the robustness of the results to settings in which the manager may incur additional costs if she does not disclose but the analyst issues a report, and to settings in which the analyst's information is less precise than the manager's information.

Our results provide potential regulatory implications, by implying that if the regulator can increase the probability of discovery of a firm's information by various third-party mechanisms, such as analyst coverage, it always has a positive effect on the information environment. Therefore, as long as actions that facilitate more discovery of firm's private information by a third party are not too costly, they are desired.

Appendix A

A1. The Minimum Principle property

A useful property of voluntary disclosure games that also holds in our model is the Minimum Principle property, first described by [Acharya et al. \(2011\)](#). We refer to this property below and thus provided it here. The minimum principle states that the price following no disclosure, $P^{\text{ND}}(\sigma)$, obtains a global minimum at the equilibrium threshold.

¹² In this model the quality of outside information depends on the probabilities g_I and g_U that the analyst observes s^a , as well as on the precision of s^a , captured by $f(s|s^a)$. For comparability with the basic setup, we treat $f(s|s^a)$ as given, and assume that changes in coverage affect only g_I and g_U .

Fact 3 (“The Minimum Principle,” Acharya et al. 2011, Proposition 1). The equilibrium threshold σ^* is the unique disclosure threshold that minimizes the price given no disclosure, that is, $\sigma^* = \min_{\sigma} P^{\text{ND}}(\sigma)$.

A2. Proof of Proposition 2

Proof. Denote by $P^{\text{ND}}(\sigma, r)$ the price given no disclosure by the firm or the analyst, as a function of a given disclosure threshold, σ , and a given analyst coverage r . $P^{\text{ND}}(\sigma, r)$ is given by (1). In addition, define $G(r, \sigma)$ as the PEF function (Eq. (5)) for a given disclosure threshold σ and analyst coverage r :

$$\begin{aligned} G(r, \sigma) &= -E[(s - P(\sigma, r))^2] \\ &= -(1 - q)(1 - g_U(r))E\left[(s - P^{\text{ND}}(\sigma, r))^2\right] \\ &\quad - q(1 - g_I(r))F(\sigma)E\left[(s - P^{\text{ND}}(\sigma, r))^2 \mid s \leq \sigma\right]. \end{aligned}$$

Note that in equilibrium the manager's disclosure threshold is $\sigma = \sigma^*(r)$ and hence, $\text{PEF}(r) = G(r, \sigma^*(r))$.

We need to show that in equilibrium, PEF is increasing in r , that is $\frac{d\text{PEF}}{dr} > 0$. This equals to

$$\begin{aligned} \frac{d\text{PEF}}{dr} &= \frac{dG(r, \sigma^*(r))}{dr} = \frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} \\ &\quad + \frac{\partial G(r, \sigma)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} \frac{d\sigma^*(r)}{dr}. \end{aligned}$$

A sufficient condition for $\frac{d\text{PEF}}{dr} > 0$ is that (1) $\frac{\partial G}{\partial r} \Big|_{\sigma=\sigma^*(r)} > 0$ and (2) $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. We prove these two properties below.

1. Proof that $\frac{\partial G}{\partial r} \Big|_{\sigma=\sigma^*(r)} > 0$:

$\frac{\partial G(r, \sigma)}{\partial r}$ is given by

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial r} &= (1 - q)g'_U(r)E\left[(s - P^{\text{ND}}(\sigma, r))^2\right] \\ &\quad + q \cdot g'_I(r) \cdot F(\sigma)E\left[(s - P^{\text{ND}}(\sigma, r))^2 \mid s \leq \sigma\right] \\ &\quad + 2(1 - q)(1 - g_U(r))E[s - P^{\text{ND}}(\sigma, r)] \\ &\quad \times \frac{\partial P^{\text{ND}}(\sigma, r)}{\partial r} \\ &\quad + 2q(1 - g_I(r))F(\sigma)E[s - P^{\text{ND}}(\sigma, r) \mid s \leq \sigma] \\ &\quad \times \frac{\partial P^{\text{ND}}(\sigma, r)}{\partial r}. \end{aligned}$$

Using (1) one can assure that

$$(1 - q)(1 - g_U(r))E[s - P^{\text{ND}}(\sigma, r)] + q(1 - g_I(r))F(\sigma) \times E[s - P^{\text{ND}}(\sigma, r) \mid s \leq \sigma] = 0,$$

and thus the last two terms sum to zero. At $\sigma = \sigma^*(r)$ we therefore obtain

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} &= (1 - q)g'_U(r)E[(s - \sigma^*(r))^2] \\ &\quad + q \cdot g'_I(r) \cdot F(\sigma^*(r)) \\ &\quad \times E[(s - \sigma^*(r))^2 \mid s \leq \sigma^*(r)]. \end{aligned}$$

Since, by definition, $g'_U(r) \geq 0$ and $g'_I(r) \geq 0$, with at least one strict inequality, we obtain $\frac{\partial G(r, \sigma)}{\partial r} \Big|_{\sigma=\sigma^*(r)} > 0$.

2. Proof that $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$:

We can rewrite $G(r, \sigma)$ as

$$\begin{aligned} G(r, \sigma) &= -(1 - q)(1 - g_U(r)) \\ &\quad \times \int_0^1 (s - P^{\text{ND}}(\sigma, r))^2 f(s) ds \\ &\quad - q(1 - g_I(r)) \int_0^{\sigma} (s - P^{\text{ND}}(\sigma, r))^2 f(s) ds. \end{aligned}$$

Differentiating with respect to σ we obtain

$$\begin{aligned} \frac{\partial G(r, \sigma)}{\partial \sigma} &= -2(1 - q)(1 - g_U(r)) \\ &\quad \times \int_0^1 (s - P^{\text{ND}}(\sigma, r))f(s) ds \\ &\quad \times \left(-\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma}\right) \\ &\quad - q(1 - g_I(r))2 \int_0^{\sigma} (s - P^{\text{ND}}(\sigma, r))f(s) ds \\ &\quad \times \left(-\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma}\right) \\ &\quad - q(1 - g_I(r))(\sigma - P^{\text{ND}}(\sigma, r))^2. \quad (\text{A.1}) \end{aligned}$$

To obtain $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)}$ observe that: (i) by Fact 1, $\sigma^*(r) = P^{\text{ND}}(\sigma^*(r), r)$. Thus, the third term in (A.1) equals zero; and (ii) by the minimum principle (Fact 3), $\frac{\partial P^{\text{ND}}(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. Therefore, the first two terms in (A.1) also equal zero. Thus $\frac{\partial G}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. \square

A3. Prices and disclosure in the extended model with a trading stage

This appendix provides additional technical results for the extended model that is described in Section 5, which includes a disclosure stage that is followed by a trading stage.

A3.1. Prices and the bid-ask spread

In this section we provide a short derivation of the bid and ask prices and the resulting bid-ask spread in a standard static GM setting. Readers who are familiar with this derivation can skip directly to Lemma 3.

First note that a strategic uninformed trader never trades. Such a trader understands that since the market maker breaks even, and an informed trader gains an information rent, an uninformed trade is expected to generate a loss. Moreover, a strategic informed trader always buys if $x = 1$ and sells if $x = 0$. This is because the public belief in the beginning of the trading stage, μ , is between zero and one, and thus the bid and ask prices are also between zero and one.¹³ Given that the informed strategic trader always trades, it is clear that no trade does not convey additional

¹³ For simplicity, assume that in the zero probability events that there is no uncertainty about x in the beginning of the trading stage, that is, $s = \mu = 1$, and $s = \mu = 0$, the informed trader still chooses to buy and sell, respectively, for a fair price.

information on the asset’s value. Therefore, the posterior belief following no trade is $E[\tilde{x} \mid \mu, \text{no trade}] = \mu$.

Let “purchase” and “sale” denote the events where the trader purchases or sells one unit, respectively. For a given public belief μ , the probability of a “purchase” event is $p\chi\mu + (1-p)\frac{1}{2}$. Conditional on a purchase event, the probability that the trader is informed is $\Pr(\text{informed} \mid \text{purchase}) = \frac{p\chi\mu}{p\chi\mu + (1-p)\frac{1}{2}}$. Thus, the market maker sets an ask price that equals

$$a(\mu) \equiv E[\tilde{x} \mid \mu, \text{purchase}] = \frac{p\chi\mu}{p\chi\mu + (1-p)\frac{1}{2}} \cdot 1 + \frac{(1-p)\frac{1}{2}}{p\chi\mu + (1-p)\frac{1}{2}} \cdot \mu = \frac{1-p+2p\chi}{1-p+2p\chi} \mu. \tag{A.2}$$

A similar calculation results in a bid price of

$$b(\mu) \equiv E[\tilde{x} \mid \mu, \text{sale}] = \frac{1-p}{1-p+2p\chi(1-\mu)} \mu. \tag{A.3}$$

It is easy to see that $b(\mu) < \mu < a(\mu)$ for any $\mu \in (0, 1)$ and $p \in (0, 1)$, and that both $a(\mu)$ and $b(\mu)$ are strictly increasing in μ . We can use (A.2) and (A.3) to calculate the bid-ask spread $\Psi(\mu) \equiv a(\mu) - b(\mu)$. The following lemma provides some properties of the bid-ask spread.

Lemma 3. The bid-ask spread, $\Psi(\mu)$, has the following properties:

1. It is a strictly concave inverse U-shape function of μ .
2. $\Psi(0) = \Psi(1) = 0$.
3. For any $\mu \in (0, 1)$, the spread is increasing in p and χ .

The proof is trivial and merely involves differentiation of (6) after substituting (A.2) and (A.3), and thus is omitted. The main characteristic of the bid-ask spread that we will be using is the concavity in the beliefs, μ .

A3.2. Disclosure decision in the extended model

In this section we analyze the manager’s disclosure strategy when she knows that a trading stage occurs following her disclosure decision. The basic model in Section 3 assumes risk neutral pricing based on all publicly available information, that is, assumes $P = \mu$. In the extended model, however, there are three possible prices: an ask price $a(\mu)$ when the trader buys one unit (a “purchase”), a bid price $b(\mu)$ when the trader sells one unit (a “sale”), and μ when there is no trade. From an outsider’s point of view, such as the market maker, the expected price is always μ . This can be easily seen using the law of iterated expectation:

$$E[P; \mu] = \Pr(\text{purchase}; \mu) \cdot a(\mu) + \Pr(\text{sale}; \mu) \cdot b(\mu) + \Pr(\text{no trade}; \mu) \cdot \mu = E[\tilde{x} \mid \mu] = \mu.$$

If an informed manager chooses to disclose her signal s , then this leads to a public belief $\mu = s$. Following disclosure, because the manager has the same information as the market maker and the public regarding the value of the firm, the informed manager expected price, or payoff, is also $U^D(s) \equiv E[P; s] = s$. This is not the case, however,

if neither the manager nor the analyst disclose. In such a case an informed manager has a better prediction than the market maker about the information of the informed trader, and thus about the probabilities of purchase and sale events. A manager with a better signal s , is more optimistic about the possibility that the trader will purchase and the price will be $a(\mu)$, and gives a lower probability to a price of $b(\mu)$. Thus, in contrast to the basic model, the informed manager’s payoff conditional on no disclosure is increasing in her type.

Nevertheless, one can show that the extended model has a threshold equilibrium and, moreover, this threshold is the same as the one in the basic model. The following proposition describes the equilibrium of the extended two-stage model.

Proposition 4. The unique equilibrium of the extended model has a threshold disclosure strategy, σ^ . The threshold σ^* is the unique solution of the indifference condition (2), as in the basic model.*

Proof. Let the public expectation of \tilde{x} given no disclosure be some exogenous belief $\mu = P^{\text{ND}}$. We prove the proposition using the following steps:

1. **Type $s = P^{\text{ND}}$ is indifferent:** A manager that observes a signal s and expects a public belief of P^{ND} , expects a payoff of

$$U^{\text{ND}}(s, P^{\text{ND}}) \equiv \Pr(\text{purchase}; s) \cdot a(P^{\text{ND}}) + \Pr(\text{sale}; s) \cdot b(P^{\text{ND}}) + \Pr(\text{no trade}; s) \cdot \mu. \tag{A.4}$$

Due to the law of iterated expectations, $U^{\text{ND}}(s, s) = s = U^D(s)$, so an informed manager with a signal $s = P^{\text{ND}}$ is indifferent whether to disclose or not.

2. **Equilibrium involves a threshold strategy:** From the analysis in A.3.1 we know that a manager with a signal s expects the following probabilities of events:

$$\Pr(\text{purchase}; s) = p\chi s + \frac{1-p}{2}$$

$$\Pr(\text{sale}; s) = p\chi(1-s) + \frac{1-p}{2}$$

$$\Pr(\text{no trade}) = p(1-\chi).$$

Substituting these probabilities in (A.4) we can easily see that

$$\frac{\partial U^{\text{ND}}(s, \cdot)}{\partial s} = p\chi \Psi(P^{\text{ND}}),$$

where $\Psi(P^{\text{ND}})$ is defined using (6). Because, by definition, $\Psi \leq 1$, then $\frac{\partial U^{\text{ND}}(s, \cdot)}{\partial s} \in (0, 1)$ for any s . Thus, given step 1, $U^{\text{ND}}(s, P^{\text{ND}}) \leq U^D(s) = s$ if and only if $s \leq P^{\text{ND}}$. That is, there is a threshold equilibrium. Moreover, let σ^* denote the equilibrium threshold, then $\sigma^* = P^{\text{ND}}$.

3. **Threshold calculated using the same condition as in the basic model:** Finally, given that there is a threshold equilibrium, in equilibrium the belief following no disclosure by the manager or the analyst satisfies (1), and given step 2 the threshold type is a solution to the fixed point condition (2). This is the same condition as in the basic model and therefore the threshold is the same.

□

Proposition 4 entails that the threshold is independent of χ , the probability that the trader is informed, and is the same as the threshold in a disclosure game where prices simply equal to the expected fundamental. Therefore, all the results of **Section 3**, including **Proposition 1** about the effect of changes in analyst coverage, hold in this model as well. To see the intuition behind the result recall that in the basic model the public belief following no disclosure is $\mu = \sigma^* = P^{ND}(\sigma^*)$ (**Fact 1**). Thus, at the beginning of the trading stage, an informed trader of type σ^* has the same beliefs as the market, whether the signal was disclosed or not. Thus, this type expects an average price of $\mu = \sigma^*$ following disclosure as well as following no disclosure.

A4. Proof of Proposition 3

Proof. For a given and constant value of q , define a function $H(r, \sigma)$ that equals the expected spread conditional on analyst coverage r and a given disclosure threshold σ (which may not be the equilibrium threshold), as follows:

$$H(\sigma, r) \equiv \text{Pr ND}(r, \sigma) \Psi(P^{ND}(\sigma, r)) + ((1 - q)g_U(r) + q \cdot g_I(r)) \cdot E[\Psi(s)] + q \cdot (1 - g_I(r)) \int_{\sigma}^1 \Psi(s) \cdot f(s) ds, \tag{A.5}$$

where

$$\text{Pr ND}(r, \sigma) \equiv (1 - q)(1 - g_U(r)) + q(1 - g_I(r))F(\sigma) \tag{A.6}$$

is the probability of no disclosure, and $P^{ND}(\sigma, r)$, given in (1), is the price following no disclosure by the manager or the analyst. When evaluated at the equilibrium disclosure threshold, $H(\sigma, r)$ is our measure of illiquidity, that is, $IL(q, r) = H(\sigma^*(r), r)$. Thus, the total derivative of $IL(q, r)$ with respect to r is:

$$\frac{dIL(q, r)}{dr} = \frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} + \frac{\partial H(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} \frac{d\sigma^*(r)}{dr}. \tag{A.7}$$

To obtain $\frac{dIL(q, r)}{dr} < 0$ it is sufficient to show that $\frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} < 0$ and $\frac{\partial H(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. We establish these sufficient conditions in the two lemmas below.

Lemma 4. $\frac{\partial H(\sigma, r)}{\partial r} \Big|_{\sigma=\sigma^*(r)} < 0$.

Proof. We show that $\frac{\partial H(\sigma, r)}{\partial r} < 0$ for any given σ , and hence it also holds for $\sigma = \sigma^*(r)$. Given the continuity of $H(r, \sigma)$ in r , it is sufficient to show that $H(r_h, \sigma) < H(r_l, \sigma)$ for any $r_h > r_l$ and any σ .

1. Using (A.10), we compute $H(r_l, \sigma) - H(r_h, \sigma)$:

$$\begin{aligned} &H(r_l, \sigma) - H(r_h, \sigma) \\ &= \text{Pr ND}(r_l, \sigma) \Psi(P^{ND}(\sigma, r_l)) \\ &\quad - \text{Pr ND}(r_h, \sigma) \Psi(P^{ND}(\sigma, r_h)) \\ &\quad + [(1 - q)(g_U(r_l) - g_U(r_h)) \\ &\quad + q \cdot (g_I(r_l) - g_I(r_h))] \cdot E[\Psi(s)] \end{aligned}$$

$$\begin{aligned} &+ q \cdot (g_I(r_h) - g_I(r_l)) \int_{\sigma}^1 \Psi(s) \cdot f(s) ds \\ &= \text{Pr ND}(r_l, \sigma) \Psi(P^{ND}(\sigma, r_l)) \\ &\quad - \text{Pr ND}(r_h, \sigma) \Psi(P^{ND}(\sigma, r_h)) \\ &\quad - (1 - q)(g_U(r_h) - g_U(r_l)) \cdot E[\Psi(s)] \\ &\quad - q \cdot (g_I(r_h) - g_I(r_l))F(\sigma) \cdot E[\Psi(s) | s < \sigma]. \end{aligned}$$

We can therefore establish that $H(r_l, \sigma) - H(r_h, \sigma) > 0$ if and only if

$$\begin{aligned} &\text{Pr ND}(r_l, \sigma) \Psi(P^{ND}(\sigma, r_l)) \\ &> \text{Pr ND}(r_h, \sigma) \Psi(P^{ND}(\sigma, r_h)) \\ &\quad + (1 - q)(g_U(r_h) - g_U(r_l)) \cdot E[\Psi(s)] \\ &\quad + q \cdot (g_I(r_h) - g_I(r_l))F(\sigma) \cdot E[\Psi(s) | s < \sigma]. \tag{A.8} \end{aligned}$$

2. Now observe from (1) that

$$\begin{aligned} &\text{Pr ND}(r, \sigma) \cdot P^{ND}(\sigma, r) = (1 - q)(1 - g_U(r))E[s] \\ &\quad + qF(\sigma)(1 - g_I(r)) \cdot E[s | s < \sigma]. \end{aligned}$$

This equation, applied to r_l and r_h , together with some algebra, leads to

$$\begin{aligned} &\text{Pr ND}(r_l, \sigma) \cdot P^{ND}(\sigma, r_l) = \text{Pr ND}(r_h, \sigma) \cdot P^{ND}(\sigma, r_h) \\ &\quad + (1 - q)(g_U(r_h) - g_U(r_l))E[s] \\ &\quad + q(g_I(r_h) - g_I(r_l))F(\sigma) \cdot E[s | s < \sigma]. \tag{A.9} \end{aligned}$$

Observe the similarity between (A.8) and (A.9); in the next step we use (A.9) to prove that the inequality in (A.8) indeed holds.

3. We can use (A.6) to rewrite (A.9) explicitly as

$$\begin{aligned} &P^{ND}(\sigma, r_l) = A \cdot P^{ND}(\sigma, r_h) + B \cdot E[s] \\ &\quad + (1 - A - B) \cdot E[s | s < \sigma] \end{aligned}$$

where $A = \frac{\text{Pr ND}(r_h, \sigma)}{\text{Pr ND}(r_l, \sigma)}$ and $B = \frac{(1 - q)(g_U(r_h) - g_U(r_l))}{\text{Pr ND}(r_l, \sigma)}$. This representation presents $P^{ND}(\sigma, r_l)$ as an average of $P^{ND}(\sigma, r_h)$ and various signals. In order to obtain (A.8) remember that $\Psi(\cdot)$, is a strictly concave function (**Lemma 3**). Thus, by definition,

$$\begin{aligned} &\Psi(P^{ND}(\sigma, r_l)) < A \cdot \Psi(P^{ND}(\sigma, r_h)) + B \cdot E[\Psi(s)] \\ &\quad + (1 - A - B) \cdot E[\Psi(s) | s < \sigma]. \end{aligned}$$

This inequality is simply (A.8), and thus implies that $H(r_l, \sigma) > H(r_h, \sigma)$. □

Lemma 5. $\frac{\partial H}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$.

Proof. Differentiating (A.5) with respect to σ we obtain

$$\begin{aligned} \frac{\partial H}{\partial \sigma} &= q(1 - g_I(r))f(\sigma) [\Psi(P^{ND}(\sigma, r)) - \Psi(\sigma)] \\ &\quad + \text{Pr ND}(r, \sigma) \Psi'(\cdot) \frac{\partial P^{ND}}{\partial \sigma}. \tag{A.10} \end{aligned}$$

To obtain $\frac{\partial H}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)}$ observe that: (i) by **Fact 1**, $\sigma^*(r) = P^{ND}(\sigma^*(r), r)$. Thus, the first term in (A.10) equals zero; and (ii) by the minimum principle (**Fact 3**), $\frac{\partial P^{ND}(\sigma, r)}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. Therefore, the second term in (A.10) also equals zero. Thus $\frac{\partial H}{\partial \sigma} \Big|_{\sigma=\sigma^*(r)} = 0$. □

This completes the proof of **Proposition 3**. □

A5. Proof of Lemma 2

Proof. The paragraph before the lemma explains why any equilibrium must have a threshold strategy. For a given disclosure threshold σ , define $\tilde{C}(\sigma) \equiv C(q^{AO}(\sigma))$ where $q^{AO}(\sigma)$ is defined in (8). It is easy to observe from (8) that $q^{AO}(0) = 0$ and $\frac{\partial q^{AO}}{\partial \sigma} > 0$. Given that $C(0) = 0$ and $C' > 0$, then $\tilde{C}(0) = 0$ and $\tilde{C}' > 0$.

For a given disclosure threshold σ , define

$$J(\sigma) \equiv P^{ND}(\sigma) - \frac{g_I}{1 - g_I} \tilde{C}(\sigma),$$

where $P^{ND}(\sigma)$ is defined in (1). By Eq. (7), the equilibrium threshold σ_C^* is defined using the equality $\sigma_C^* = J(\sigma_C^*)$. To see that such a threshold exists, first observe from (1) that $P^{ND}(0) = P^{ND}(1) = E[s]$. Thus, $J(0) = E[s] > 0$ and $J(1) < P^{ND}(1) = E[s] < 1$. By continuity, σ_C^* exists. Moreover, it is strictly greater than zero, which is Part 2 of the lemma.

Part 1 of the lemma is immediate by the fact that, if $g_I > 0$, then $J(\sigma) < P^{ND}(\sigma)$, and thus $\sigma_C^* < \sigma^*$, where σ^* is defined in Fact 1.

By the minimum principle (Fact 3), the function $P^{ND}(\sigma)$ is decreasing for values $\sigma < \sigma^*$. Because $\tilde{C}' > 0$, $J(\sigma)$ is also decreasing for $\sigma < \sigma^*$. This assures the uniqueness of the threshold. \square

A6. Price efficiency in a model with noisy analyst's signal

Consider the model described in Section 6.2, in which the analyst's information is less precise than the manager's information. Specifically, assume that the analyst may observe a noisy signal s^a about s , and that s^a , if observed, is published before an informed manager decides whether to disclose s or not. In what follows, we treat the probability that the manager is informed q , as well as the distributions of s and s^a as given and fixed, and consider only a change in the conditional probabilities that the analyst is informed, g_I and g_U .

General analysis of price efficiency

The purpose of this section is to analyze how price efficiency, as defined in Eq. (5), behaves in this model. First, consider a game in which $g_U = g_I = 0$, that is, there is no analyst. This game is the model of Dye (1985). For a given probability that the manager is informed q , let $PEF_0(q)$ be the price efficiency in this game. Now consider the game in which $g_U = g_I = 1$, that is, s^a is always publicly available. Following a given realization of s^a , the game is similar to the model of Dye (1985) with a posterior probability $f(s|s^a)$. The manager, if informed, decides whether to disclose using a threshold strategy $\sigma^*(s^a)$. Let $PEF_1(q)$ be the ex ante price efficiency in this game, that is, $PEF_1(q)$ is a weighted average of price efficiencies that are calculated for any given signal s^a .

In a model with general g_U and g_I , denote by $Pr s^a \equiv q \cdot g_I + (1 - q)g_U$ the overall probability that the analyst observes s^a and publishes a report, and by

$$\hat{q}_1 = \frac{q \cdot g_I}{(1 - q)g_U + q \cdot g_I}, \text{ and}$$

$$\hat{q}_0 = \frac{q(1 - g_I)}{(1 - q)(1 - g_U) + q(1 - g_I)}$$

the probabilities that the manager is informed conditional on an analyst report, and conditional on no analyst report, respectively. Price efficiency equals to

$$PEF(q, g_I, g_U) = (1 - Pr s^a)PEF_0(\hat{q}_0) + Pr s^a \cdot PEF_1(\hat{q}_1). \tag{A.11}$$

This is simply a result of the law of iterated expectation.

Using (A.11) we can analyze how price efficiency is affected by a small increase in coverage, that is, an increase in g_I and/or g_U (remember we assume that $g'_U(r) \geq 0$ and $g'_I(r) \geq 0$). The effect of an increase in coverage can be decomposed into two parts:

1. A direct change: an increase in the probability of an analyst report $Pr s^a$, that increases the relative weight of $PEF_1(\hat{q}_1)$ and decreases the weight of $PEF_0(\hat{q}_0)$.
2. An indirect change: changes in \hat{q}_1 and \hat{q}_0 that affect the manager's disclosure strategy and change $PEF_1(\hat{q}_1)$ and $PEF_0(\hat{q}_0)$, respectively.

Uncorrelated information endowment ($g_U = g_I$)

If $g_I(r) = g_U(r) = g(r)$, then $\hat{q}_0 = \hat{q}_1 = q$ and $Pr s^a = g$. Thus, a change in r affects price efficiency only through a direct change in $Pr s^a$ (effect 1 above). Overall price efficiency therefore increases if and only if $PEF_1(q) > PEF_0(q)$.

We first show this is always the case when s^a and s follow a joint normal distribution. Without loss of generality assume that both have zero mean, that is, $s^a = s + u$ where $s \sim N(0, \sigma_s^2)$, $u \sim N(0, \sigma_u^2)$, and $cov(s, u) = 0$. Thus, $s | s^a \sim N(as^a, b^2\sigma_s^2)$ where $a = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_u^2}$ and $b = 1 - a < 1$. Let z^* be the disclosure threshold in a Dye model when the prior is $N(0, 1)$. Proposition 2 in Acharya et al. (2011) shows that when the prior is distributed $N(\mu, \sigma^2)$, the disclosure threshold is $\mu + \sigma z^*$. Thus, for normal distributions, price efficiency (as defined in (5)) satisfies $PEF_{N(\mu, \sigma^2)} = \sigma^2 PEF_{N(0, 1)}$. An immediate implication is that $PEF_1(q) = b^2 PEF_0(q) > PEF_0(q)$.

In the general case we can show that $PEF_1(q) \geq PEF_0(q)$ using an argument that follows Hart et al. (2017). We describe the argument informally and point the reader to Hart et al. (2017) for the formal treatment. Consider the game where the analyst always publishes a report, that is, $g_I = g_U = 1$, and suppose that, instead of risk neutral pricing, the market ("receiver") can commit at the beginning of the game on any pricing function. Specifically, suppose that the market chooses to ignore the signal s^a : $P = s$ following a disclosure by the manager, and $P = E[s | ND]$ following no such disclosure, where this price is the same as the price in a game without an analyst. Clearly, following such commitment the manager will choose the same disclosure strategy as in a game without an analyst, and price efficiency will be PEF_0 . The main result of Hart et al. (2017) is that such a commitment cannot decrease the quadratic loss; that is, price efficiency without such commitment, PEF_1 , is equal to or greater than PEF_0 .

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