

# DISPERSED INFORMATION, STICKY PRICES AND MONETARY BUSINESS CYCLES: A HAYEKIAN PERSPECTIVE <sup>\*</sup>

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## Abstract

We study the propagation of nominal shocks in a dispersed information economy where firms learn from and respond to information generated by their activities in product and factor markets. We prove the existence of a “Hayekian benchmark”, defined by conditions under which imperfect information has *no* effect on equilibrium outcomes. This occurs under fairly general conditions when prices are flexible, i.e. without nominal frictions, informational frictions are irrelevant. With sticky prices, however, this irrelevance obtains only if there are no strategic complementarities in pricing and aggregate and idiosyncratic shocks are equally persistent. With complementarities and/or differences in persistence, the interaction of nominal and informational frictions slows down price adjustment, amplifying real effects from nominal shocks (relative to a full information model with only nominal frictions). In a calibrated model, the amplification is most pronounced over the medium to long term. In the short run, market generated information leads to substantial aggregate price adjustment, even though firms may be completely unaware of changes in aggregate conditions.

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# 1 Introduction

Models of the monetary transmission mechanism often attribute an important role to adjustment frictions for prices and wages in order to account for delays and inefficiencies in how markets respond to macroeconomic shocks. These frictions come either in the form of physical adjustment costs or sticky prices, or in the form of limits to the available information, and rich literatures have developed around each of these themes.<sup>1</sup>

In this paper, we seek to offer a novel perspective on the propagation of nominal shocks, which puts at its center the interaction between these two sources of frictions, but also takes seriously the informational role of markets and prices in coordinating economic outcomes that was emphasized by Hayek (1945). Our point of departure is a standard model of nominal price-setting by imperfectly competitive firms that are subject to market-specific and aggregate shocks to demand and costs. Their own activities in product and factor markets are the most immediate sources of information. But since these markets reflect the combined effects of these shocks, they naturally induce imperfect and heterogeneous beliefs about aggregate conditions. We assess the impact of informational frictions by comparing the equilibrium allocations of an economy in which all economic shocks are commonly known with those of an otherwise identical economy in which the signals generated by their market activities are the firms' *only* source of information.

We start with an environment where this form of limited information is the only constraint on firms' pricing decisions. In other words, firms are unconstrained in their ability to adapt prices to the information available from their market activities, specifically their own sales and wages. We obtain a striking result: firms set prices in a fashion that replicates the full information outcome. Imperfect information is thus completely irrelevant for equilibrium allocations. This result is based on two simple insights. First, in any Bayesian game, the equilibrium outcome that obtains with perfect information remains an equilibrium outcome with imperfect information, whenever the

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<sup>1</sup>Both strands are too large to fully survey here, so we restrict ourselves to a few recent, representative examples. Nominal rigidities have been studied extensively - a few notable recent contributions on their quantitative significance include Ball and Romer (1990), Rotemberg and Woodford (1997), Christiano et al. (2005), Eichenbaum et al. (2011), Golosov and Lucas (2007), Nakamura and Steinsson (2008) and Midrigan (2011). Informational frictions as a source of monetary non-neutrality dates back at least to the seminal work of Phelps (1970) and Lucas (1972), but recent contributions include Woodford (2003), Mankiw and Reis (2002), Angeletos and La'O (2009; 2012) Mackowiak and Wiederholt (2009; 2010), Coibion and Gorodnichenko (2012) and Alvarez et al. (2011).

information structure is sufficiently rich to allow all agents to infer their best responses to the actions of the other players - or in our case, whenever firms are able to perfectly figure out their optimal prices, even though they may still remain highly uncertain about aggregate conditions, or about the prices set by other firms. Second, the concurrent market information allows the firms to do just that. To see why, note that, in the absence of nominal frictions, firms' pricing decisions are based on a static trade-off between marginal costs and revenues. The signals the firms obtain from their transactions in input and output markets contain very accurate information about these objects, allowing firms to perfectly infer their optimal price.

We term this result the 'Hayekian benchmark', because it is reminiscent of the idea first developed by Hayek in his influential (1945) essay: "*(in) a system where knowledge of the relevant facts is dispersed, prices can act to coordinate.....The most significant fact about the system is the economy of knowledge with which it operates, how little the individual participants need to know in order to be able to take the right action.*" Hayek (1945) thus emphasizes the parsimony of knowledge with which the price system guides individual participants to take decisions that are not only in their own best interest, but ultimately lead to a socially efficient allocation of resources, despite the lack of central organization and communication of all information to market participants.<sup>2 3</sup>

Next, we consider environments with nominal adjustment frictions or sticky prices. These frictions interfere with the firm's ability to adapt its price to available information, turning the price-setting problem into a dynamic one and requiring firms to forecast future marginal costs and revenues from current signals. We show that with nominal adjustment frictions, dispersed information is relevant for equilibrium allocations only if firms have a motive to disentangle different types of idiosyncratic and aggregate shocks; otherwise, the sticky price model with dispersed information results in exactly the same equilibrium allocations as its full information counterpart. Motives for disentangling different shocks arise only if there are differences in time series properties (in particu-

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<sup>2</sup>Hayek (1945) referred to a 'price system' without being explicit about the underlying market structure. His insight is implicitly invoked in models where the information structure is left unspecified, but has not been articulated in a formal model with a well-defined market structure. Our market structure has price setting under imperfect competition, but this general insight about the effectiveness and parsimony of market information goes through almost exactly.

<sup>3</sup>This is rather different from Grossman and Stiglitz (1980) who emphasize the role of prices in aggregating and publicizing information. Our arguments do not rely on market prices being fully revealing or even public signals. In fact, firms' signals could be very poor indicators of the true nature of shocks hitting the economy, but, as long as they provide an accurate indication of the firm's individually optimal decisions, the Hayekian benchmark obtains.

lar, the persistence) of aggregate and idiosyncratic shocks or strategic complementarities in pricing decisions. The intuition is similar to the static case - in the absence of strategic complementarities and differences in persistence, firms' current market signals are sufficient for the firms' best forecast of profit-maximizing prices in future periods and through them, for their current optimal price. In such a scenario, the limited nature of the firm's information set (in the sense that it only contains its market signals) again does not have any implications for its decision. This is the dynamic analogue of the static Hayekian benchmark.

We then consider two canonical models with nominal frictions – Calvo pricing and menu costs – to explore how pricing complementarities and differential degrees of persistence between idiosyncratic and aggregate shocks interact to generate departures from the Hayekian benchmark. For tractability and expositional clarity, we focus on the empirically plausible limiting case when aggregate shocks are small in comparison to idiosyncratic shocks.

We show that, under dispersed information, pricing complementarities and differential degrees of persistence have mutually reinforcing effects in slowing down aggregate price adjustment. First, if idiosyncratic shocks and aggregate nominal disturbances are both permanent, then prices fully absorb the nominal shocks in the long run. When instead idiosyncratic shocks are less persistent than aggregate shocks, nominal shocks can have long-lasting (and at our small shocks limit, permanent) effects, the magnitude of which depend on the relative rates of mean reversion. Second, pricing complementarities on their own do not create long-run non-neutrality, but they amplify the effects of differential degrees of persistence. Third, complementarities play an important role in slowing down the speed with which prices adjust to aggregate nominal shocks in the short run.

The role of differential persistence is straight-forward: When nominal shocks are small, firms attribute most (and, in the small shocks limit, all) of the changes in their sales and wage signals to idiosyncratic factors. When these factors are less persistent than the aggregate shock, firms, recognizing the forward-looking nature of their pricing decision tend to make smaller adjustments, relative to a situation where the nature of the shock is perfectly observed.

The role of strategic complementarities is more subtle. The same equilibrium linkages that give rise to complementarities in pricing also determine the importance of aggregate prices for

the firm-level sales and wage signals. Since aggregate prices reflect aggregate shocks only slowly (due to the nominal friction), complementarities slow down the rate at which these shocks enter firm-level signals and hence their prices, amplifying the sluggishness in aggregate price adjustment. This informational role for complementarities is novel and quite different from the well-known strategic channel emphasized both in the full information sticky price models as well as in the dispersed information models cited in footnote 1. In these models, firms receive news that aggregate conditions have changed, but they refrain from fully adjusting to these news because they expect that other firms will not fully adjust either - and hence dampen their responses.<sup>4</sup> In our model, and especially when aggregate shocks are very small, the strategic channel is practically absent because firms attach almost zero probability to a change in aggregate conditions. The informational channel instead is sufficiently powerful to generate even more sluggishness than the strategic channel in a full information model.

Finally, we explore the quantitative magnitude of this interaction between nominal and informational frictions by calibrating models with Calvo pricing and menu costs to match moments of micro data on price adjustment. Under this calibration, dispersed information slows down price adjustment significantly but mostly at medium to long horizons. Nevertheless, the Hayekian mechanism remains quite powerful, since a significant proportion of aggregate shocks is absorbed by prices in the long run, even though firms never learn the true nature of the aggregate shock. On the other hand, the two economies display similar responses to aggregate shocks over the first 3-4 quarters following impact, suggesting that the contribution of informational frictions to short run monetary non-neutrality is quite modest.<sup>5</sup>

Our work bears a direct relation to a large and growing work using models with heterogeneous information to study business cycles.<sup>6</sup> Much of this literature tends to model information as

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<sup>4</sup>With full information, the complementarity propagates lack of adjustment from firms that have their prices fixed to firms who have the ability to adjust, while in the dispersed information models, the complementarity amplifies an initial lack of adjustment to available information because firms are not exactly certain what other firms have observed. At its core the two mechanisms are very similar.

<sup>5</sup>As robustness checks, we also consider a variation of our model with fat-tailed idiosyncratic shocks as in Midrigan (2011). This dampens the so-called selection effect, reducing the impact of short run price adjustment and hence reinforcing the informational role of pricing complementarities. We also consider a variation where aggregate shocks are serially correlated and show how in this case the informational channel of complementarities becomes more powerful in delaying short run price adjustment.

<sup>6</sup>Besides the works cited in footnote 1, Hellwig (2005), Angeletos and La'O (2010), Amador and Weill (2010), Lorenzoni (2009), Hellwig and Veldkamp (2009), Nimark (2008), Paciello and Wiederholt (2014), Graham and Wright

abstract, noisy signals of the exogenous shocks.<sup>7</sup> Therefore, market prices and allocations do not have any part to play in the aggregation and transmission of information, ruling out the Hayekian mechanism by assumption. Our results, especially the static benchmark, highlight the central role of this assumption in generating meaningful effects from informational frictions in those papers. The presence of market-generated information makes it much harder to make information frictions relevant - in the price-setting context, we need both additional (i.e. nominal) frictions as well as assumptions about the structure of preferences/production and shocks hitting the economy.<sup>8</sup>

Our analysis builds on the canonical New Keynesian framework widely used to study the dynamics of price adjustment. Our work complements this literature by analyzing the interaction of informational frictions with various assumptions about nominal rigidity, including both time-dependent, as in Taylor (1980) or Calvo (1983), and state-dependent models, as in Caplin and Leahy (1991) and Golosov and Lucas (2007). Gorodnichenko (2010) analyzes a similar model with incomplete information and nominal frictions, but focuses on externalities affecting nominal adjustment through information acquisition. In his paper, the prospect of learning from market prices reduces firms' incentives to acquire costly information. At our Hayekian benchmark, this trade-off can be particularly extreme - markets provide firms with all the information they need, so additional information is worthless and will not be acquired at a positive cost in equilibrium. Alvarez et al. (2011) also study the interaction of nominal and informational frictions, but they introduce additional, exogenous observation costs, limiting the Hayekian channel at the heart of this paper.

Finally, our calibration draws on recent work documenting price adjustment at the micro level using large scale data sets of individual price quotes. The moments we target - the cross-sectional dispersion and time series properties of prices as well as the frequency and magnitude of price changes - are taken from the work of Bils and Klenow (2004), Nakamura and Steinsson (2008),

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(2010), Melosi (2014) and Atolia and Chahrour (2013) have also contributed to this literature.

<sup>7</sup>Important exceptions are Amador and Weill (2010), Hellwig and Venkateswaran (2009) Graham and Wright (2010) and Atolia and Chahrour (2013). Mackowiak and Wiederholt (2009) also consider endogenous signals in an extension to their baseline model.

<sup>8</sup>Our results also have implications for another important branch of the dispersed information literature - one that studies the welfare effects of additional information. In Morris and Shin (2002), Hellwig (2005) and Angeletos and Pavan (2007), additional information can reduce social welfare, due to misalignment of social and private incentives for coordination. Our analysis suggests that the applicability of these insights to market economies crucial depends on departures from the conditions characterizing the Hayekian benchmark.

Klenow and Krystov (2008), Burstein and Hellwig (2007) and Midrigan (2011).

The rest of the paper proceeds as follows. Section 2 introduces the model and establishes the Hayekian benchmark results, both with and without nominal adjustment frictions. Section 3 analyzes departures from the Hayekian benchmark in the case with Calvo pricing. Section 4 provide similar results for a calibrated version of a menu cost model with dispersed information. Section 5 presents some robustness checks and extensions, and is followed by a brief conclusion.

## 2 Model

In this section, we lay out a dynamic stochastic general equilibrium model, where firm pricing decisions are constrained by both information and nominal frictions. In order to keep the focus squarely on the impact of these two frictions, we deliberately keep the household side of the economy as close to the New Keynesian benchmark as possible. In particular, we assume that there is a representative household with perfect information about aggregate and market-specific shocks as well as access to complete contingent claims markets.<sup>9</sup>

**Production:** The economy has a single final good, which is produced by a *fully informed*, perfectly competitive firm using a continuum of intermediate goods:

$$Y_t = \left( \int B_{it}^{\frac{1}{\theta}} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where  $B_{it}$  is an idiosyncratic demand shock for intermediate good  $i$  and  $\theta > 1$  denotes the elasticity of substitution. Optimization by the final goods producer implies the usual demand function

$$Y_{it} = B_{it} Y_t \left( \frac{P_{it}}{P_t} \right)^{-\theta}, \quad (1)$$

where the aggregate price level  $P_t$  is given by

$$P_t = \left( \int B_{it} P_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (2)$$

Each intermediate good is produced with labor of type  $i$  as the sole input, according to a decreasing returns to scale production function:

$$Y_{it} = \frac{1}{\delta} N_{it}^{\frac{1}{\delta}} \quad \delta > 1.$$

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<sup>9</sup>We defer a complete description of preferences till later in this section.

Each type of labor is traded in a competitive factor market, with a market-clearing wage  $W_{it}$ .

**Information:** The information set of firm  $i$  at the time it makes its period  $t$  decision is denoted  $\mathcal{I}_{it}$ . For most of our analysis, we restrict attention to two polar cases. Under *dispersed information*, firms only observe (histories of) signals generated by their market activities - in particular, their sales  $Y_{it}$  and wages  $W_{it}$ . Under *full information*, all firms observe (histories of) all shocks, both aggregate and idiosyncratic. Obviously, these are extreme assumptions but they will permit a stark demonstration of the effect of informational assumptions on equilibrium outcomes.<sup>10</sup>

Beliefs about fundamentals (i.e. the underlying shocks) and aggregate outcomes will typically be very different in the two economies, except in the special case where the market signals allow the firm to infer the underlying shocks exactly. However, we are not interested in beliefs *per se* but in outcomes, namely prices and quantities. We will say that dispersed information is *relevant* if prices and quantities are different under dispersed information compared to an otherwise identical economy with full information. If this is not the case, i.e. prices and quantities in the two economies coincide, then we will say that dispersed information is *irrelevant*.

## 2.1 Only informational frictions: A benchmark result

We now turn to the price-setting problem of the intermediate goods producing firm. We start with an important benchmark, in which firms set prices every period, conditional on the information set described above. In other words, firms set prices subject to limited information, but face no other costs or constraints.<sup>11</sup> In this case, each firm's problem is a static one

$$\max_{P_{it}} \mathbb{E}_{it} [\lambda_t (P_{it}Y_{it} - W_{it}N_{it})] \quad , \quad (3)$$

where  $\lambda_t$  is the stochastic discount factor of the household and  $\mathbb{E}_{it}$  is the expectation conditional of  $i$ 's information set,  $\mathcal{I}_{it}$ .

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<sup>10</sup>In addition, following the heterogeneous information literature, we assume throughout that the economic structure, i.e. all structural parameters and stochastic properties of shocks, are common knowledge among firms.

<sup>11</sup>This formulation allows firms in a dispersed information economy to condition their decisions in period  $t$  on contemporaneous (i.e. period  $t$ ) market signals. Thus, there is no extra delay between the arrival of market signals and their incorporation into prices. This is the sense in which we think of the economy with only informational frictions as having 'flexible' prices. This benchmark can also be interpreted as the outcome of a game in which firms set price schedules that are functions of the market signals. Our notion of flexible prices is closely related to the one in Angeletos and La'O (2012), but, unlike that paper, we only require that prices be contingent on the firm's market information.



**Equilibrium:** An equilibrium consists of sequences of intermediate goods producers' pricing strategies  $\{P_{it}\}$  that are measurable with respect to the information set  $\mathcal{I}_{it}$  and solve (3), final good prices  $P_t$  and production choices by the final goods producer  $\{Y_{it}, Y_t\}$  that are consistent with the intermediate goods demand (1) and price index (2), and consumption and labor supply choices of the household and a pricing kernel  $\{\lambda_t\}$ , such that household choices are optimal and markets clear.

Our first result states that, in the absence of nominal frictions, informational frictions do not have *any* effect on allocations.

**Proposition 1** *Suppose firms set prices every period. Then, dispersed information is not relevant, i.e. prices and quantities under dispersed information are identical to those under full information.*

To explain the intuition behind this striking result, we proceed in two steps. We first identify conditions on the information structure  $\{\mathcal{I}_{it}\}$  under which the full information equilibrium remains sustainable with incomplete information. Specifically, we show that this is the case whenever the information set  $\mathcal{I}_{it}$  allows every firm to infer its own full information best response. This is necessary and sufficient for the full information equilibrium strategies to be feasible, and by definition of an equilibrium, the full information strategies then remain mutual best responses.

Second, we show that firms' information sets in the dispersed information economy satisfy this property. Firm  $i$ 's optimal price under full information solves the first order condition:

$$\underbrace{\Phi_1 P_{it}^{-\theta} P_t^\theta B_{it} C_t}_{\text{Marginal Revenue}} = \underbrace{\Phi_2 P_{it}^{-\theta\delta-1} (P_t^\theta B_{it} C_t)^\delta W_{it}}_{\text{Marginal Cost}} \quad (4)$$

where  $\Phi_1$  and  $\Phi_2$  are positive constants. Under dispersed information, the firm is assumed to have access to two contemporaneous signals - its own sales  $Y_{it}$  and wage rate  $W_{it}$ . From (1), the sales signal is informationally equivalent to  $P_t^\theta B_{it} C_t$ . Along with the directly observed wage signal, this gives the firm all the information it needs to accurately forecast its own marginal revenues/costs and therefore, infer its best response. By the first step, the full information equilibrium is obtained. Importantly, this conclusion is independent of the nature of the stochastic process for each of the shocks or preferences of the household, which we have so far left unspecified. Note also that this is an equilibrium in dominant strategies and hence, extremely robust to perturbations of behavior by other firms.

This result formalizes the insight from Hayek's quotation in the introduction: the information gained through market activities allows firms to align marginal costs and revenues, thereby coordinating allocations and prices on the full information outcome, even if this information remains very noisy about aggregate conditions. It is a stark demonstration of the *economy of knowledge* of the market system that Hayek was emphasizing - once individual actors incorporate the information from their market interactions, the full information allocation obtains, and any further information becomes redundant from the perspective of individual profit maximization and equilibrium allocations. We will from now on refer to the conditions under which the dispersed information is irrelevant as the "Hayekian benchmark".

This result stands in stark contrast to the findings of the rather large body of work on heterogeneous information models. For example, in Mankiw and Reis (2002), Woodford (2003), Mackowiak and Wiederholt (2009), Angeletos and La'O (2012), or other similar models, equilibrium outcomes under dispersed information are very different from their full information counterparts. The source of the difference between Proposition 1 and their environments is the underlying information structure. A common approach in the literature is to use an abstract specification - signals are modeled as arbitrary combinations of fundamental shocks and observational noise, sometimes endogenized as a choice of the firm, such as in the rational inattention framework. This rules out the Hayekian mechanism by assumption.<sup>12</sup> Proposition 1 shows that with static decisions conditioned on contemporaneous market information, this mechanism is extremely powerful.

There are a number of implications of Proposition 1. First, under these conditions, any additional information about the aggregate economy, including direct information about the shocks themselves, is irrelevant for the firm's decision. This holds irrespective of the quality or the public-versus-private content of that information. It then follows that the results in Morris and Shin (2002) or Angeletos and Pavan (2007) about the welfare implications of more information do not apply in this environment. Additional information, whether public or private, is simply irrelevant for equilibrium allocations and therefore, for welfare.

Second, the result offers a note of caution on inferences drawn from observed heterogeneity in

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<sup>12</sup>There are a few papers that use market-based signals, e.g. Lucas (1972) and Amador and Weill (2010). However, in those papers, some markets are either missing or are accessed only by a subset of agents. For example, in Amador and Weill (2010), entrepreneurs make labor supply decisions in a non-market setting, without wages to guide them.

beliefs about the relevance of informational frictions in an economy.<sup>13</sup> In the Hayekian benchmark, learning from market signals could induce a considerable amount of cross-sectional dispersion in firms' beliefs about aggregate conditions, but the extent of this confusion has no bearing on allocations. What's more, changes in cross-sectional belief dispersion that result from changes in the information structure have no bearing on aggregate outcomes, and are hence uninformative of information frictions when the conditions of the Hayekian benchmark are satisfied.

In summary, to argue that information frictions and belief dispersion affect business cycle propagation, one needs to argue for departures from the Hayekian benchmark. The conditions underlying Proposition 1 point to possible directions for such departures: nominal frictions, which turns the firm's decision problem into a dynamic one and/or delays in the arrival/observation of market-generated signals. These features interfere with the firm's ability to infer its full information best response from the market information available to it. We now impose additional structure, specifically on preferences and stochastic processes, to analyze these departures.

## 2.2 Preferences and nominal frictions

**Preferences:** The representative household maximizes its lifetime utility over consumption  $C_t$  and real balances  $M_t/P_t$ , as well as disutility of effort over a measure 1 continuum of labor types  $N_{it}$ ,

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\psi}}{1-\psi} + \ln \frac{M_t}{P_t} - \int_0^1 Z_{it} N_{it} di \right),$$

where  $\beta \in (0, 1)$ ,  $\psi > 0$ ,  $N_{it}$  and  $Z_{it}$  denote, respectively, labor supply and an idiosyncratic preference shock for labor of type  $i$ .  $\mathbb{E}_t(\cdot)$  denotes the representative household's expectations as of date  $t$ . Assuming that the household has access to a complete contingent claims market, the household's life-time budget constraint is

$$M_0 \geq \mathbb{E}_0 \sum_t \lambda_t \{ C_t P_t + i_t M_t - \int_0^1 W_{it} N_{it} di - \Pi_t - T_t \},$$

where  $\lambda_t$  denotes the economy's stochastic discount factor used to price nominal balances,  $\Pi_t$  and  $T_t$  denote aggregate corporate profits and taxes or transfers (in nominal terms),  $W_{it}$  denotes the nominal wage for labor of type  $i$ , and the term  $i_t M_t$  denotes the household's opportunity costs of

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<sup>13</sup>Some authors, e.g. Coibion and Gorodnichenko (2012) and Melosi (2012), estimate dispersed information models using forecast dispersion data from the Survey of Professional Forecasters to infer information frictions.

holding monetary balances at date  $t$ . The first order conditions of this problem are

$$\lambda_t = \beta^t \frac{C_t^{-\psi}}{P_t} = \beta^t \frac{1}{i_t M_t} = \beta^t \frac{Z_{it}}{W_{it}} = (1 + i_t) \mathbb{E}_t \lambda_{t+1} . \quad (5)$$

Along with the budget constraint and a law of motion for  $i_t$  that is determined by monetary policy, these equations characterize the solution to the household's problem. Throughout this paper, we focus on the special case where  $\ln M_t$  follows a random walk with drift  $\mu$ :

$$\ln M_{t+1} = \ln M_t + \mu + u_t ,$$

where  $u_t$  is an iid random variable, distributed  $N(0, \sigma_u^2)$ . This implies that interest rates are constant at a level  $\hat{i}$ , defined by  $(1 + \hat{i})^{-1} = \beta \mathbb{E}_t (M_t / M_{t+1}) = \beta \exp(-\mu + \sigma_u^2 / 2)$ , which we assume to be strictly positive. Along with the FOC, we can then write state prices, consumption, and wages as follows:

$$\lambda_t = \beta^t \frac{1}{M_t} \hat{i} , \quad (6)$$

$$C_t = K_0 \left( \frac{M_t}{P_t} \right)^{\frac{1}{\psi}} , \quad (7)$$

$$W_{it} = K_1 M_t Z_{it} , \quad (8)$$

where  $K_0$  and  $K_1$  are time-independent constants. Besides this characterization of the household's equilibrium behavior through static equations for aggregate consumption and type-specific wages, the constant interest rate also eliminates the effects of nominal interest rates as a public signal of aggregate economic activity.<sup>14</sup>

**Idiosyncratic shocks:** Next, we describe the stochastic processes for the preference shock  $Z_{it}$  and the demand shock  $B_{it}$ . We assume that they follow AR(1) processes (in logs) with normally distributed innovations:<sup>15</sup>

$$b_{it} = \rho_b \cdot b_{it-1} + u_{it}^b$$

$$z_{it} = \rho_z \cdot z_{it-1} + u_{it}^z$$

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<sup>14</sup>This specification closely tracks Hellwig and Venkateswaran (2009). See Hellwig (2005) and Nakamura and Steinsson (2008) for other microfounded models with similar equilibrium relationships.

<sup>15</sup>The natural logs of capital-lettered variables, are denoted by the corresponding small letters, e.g. for any variable  $X$ , we write  $x = \ln X$ .

where  $u_{it}^b, u_{it}^z$  are mean-zero, normally distributed random variables with variances  $\sigma_b^2$  and  $\sigma_z^2$ .

**Nominal frictions:** Next, we introduce nominal frictions/adjustment lags into the firm's price-setting problem. For concreteness, we consider four canonical models of nominal frictions.

- **Case I:** Prices set every period, but information observed with an N-period lag.
- **Case II:** Prices set once every N periods, but information is contemporaneous.
- **Case III:** Prices set as in Calvo, but information is contemporaneous.
- **Case IV:** Prices set subject to fixed menu costs, but information is contemporaneous.

The formal description of the firm's problem in each of the cases is below.

**Case I: Prices set every period, but information observed with a lag of N:** Here, the firm's problem is the same as (3), only the information available to the firm changes. At the time of setting period  $t$  prices, the firm has access to information N-periods old. Formally, under full information,

$$\mathcal{I}_{it}^{\text{Full}} = \{M_{t-N-s}, P_{t-N-s}, B_{it-N-s}, Z_{it-N-s}\}_{s=0}^{\infty} ,$$

and under dispersed information,

$$\mathcal{I}_{it}^{\text{Disp}} = \{Y_{it-N-s}, W_{it-N-s}\}_{s=0}^{\infty} .$$

In the other three cases, the firms' decision problem changes to account for price adjustment frictions, but information sets are the same as in the version without nominal frictions, i.e.:

$$\mathcal{I}_{it}^{\text{Full}} = \{M_{t-s}, P_{t-s}, B_{it-s}, Z_{it-s}\}_{s=0}^{\infty} \quad \text{and} \quad \mathcal{I}_{it}^{\text{Disp}} = \{Y_{it-s}, W_{it-s}\}_{s=0}^{\infty} .$$

**Case II: Prices set once every N periods:** In every reset period  $t$ , the firm solves

$$\max_{P_{it}} \mathbb{E}_{it} \sum_{s=0}^N \beta^s [\lambda_{t+s} (P_{it} Y_{it+s} - W_{it+s} N_{it+s})] . \quad (9)$$

**Case III: Prices set as in Calvo (1983):** Every period, with probability  $\xi$ , the firm can change its price. This probability is independent over time and across firms. Thus, the probability

that the firm's price remains unchanged for exactly  $T$  periods is given by  $(1 - \xi)^{T-1}\xi$ . In every reset period, the firm solves

$$\max_{P_{it}} \mathbb{E}_{it} \sum_{T=1}^{\infty} (1 - \xi)^{T-1}\xi \sum_{s=0}^{T-1} \beta^s [\lambda_{t+s} (P_{it}Y_{it+s} - W_{it+s}N_{it+s})] . \quad (10)$$

**Case IV: Prices set subject to fixed menu costs:** Let  $V(P_{it-1}, \mathcal{I}_{it})$  denote the value of firm which starts period  $t$  with a price  $P_{it-1}$  and an information set  $\mathcal{I}_{it}$ . The Bellman equation below characterizes  $V(\cdot)$ :

$$V(P_{it-1}, \mathcal{I}_{it}) = \max \left\{ \mathbb{E}_{it}[\lambda_t \Pi(P_{it-1}, M_t, P_t, B_{it}, Z_{it}) + \beta V(P_{it-1}, \mathcal{I}_{it+1})], \right. \\ \left. \max_P \mathbb{E}_{it}[\lambda_t \Pi(P, M_t, P_t, B_{it}, Z_{it}) - \lambda_t W_{it} \mathcal{C} + \beta V(P, \mathcal{I}_{it+1})] \right\} , \quad (11)$$

where  $\Pi(P, \cdot) = PY_{it} - W_{it}N_{it}$  and  $\mathcal{C}$  is a fixed cost, in terms of labor units, of changing prices.

### 2.3 Both information and nominal frictions: A dynamic irrelevance result

In this subsection, we discuss a dynamic version of our information irrelevance result for the four cases laid out above. Our notion of irrelevance is the same as before. For each case, we compare the behavior of the dispersed information economy to an identical economy subject to the same nominal friction but under full information (i.e. assuming the realizations of the underlying shocks are common knowledge). Recall that in the absence of nominal frictions, dispersed information turned out to be irrelevant because firms' signals allowed them to perfectly infer their (marginal) revenues and costs. Two complications arise in extending that logic to a dynamic environment. First, firms now have to forecast future revenues and costs using current signals.<sup>16</sup> Second, profits are weighted by an aggregate stochastic discount factor,  $\lambda_t$ . For the informational friction to be irrelevant, both forecasts of future profits and their relative weight in the firm's objective (determined by expected  $\frac{\lambda_{t+s}}{\lambda_t}$ ) must be the same under dispersed and full information.

We start by deriving conditions under the irrelevance result of Proposition 1 survives the introduction of nominal frictions. These conditions will be more stringent, but they deliver a general insight - nominal frictions make dispersed information relevant if they introduce motives to disentangle different types of shocks. This occurs when (a) aggregate and idiosyncratic shocks differ in

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<sup>16</sup>Or equivalently, forecast current revenues/costs using past signals.

their persistence or (b) general equilibrium linkages generate a direct link between aggregate prices affect firm-level revenues or costs (or, in the language of the pricing literature, there are strategic complementarities in pricing decisions).

The following proposition, the dynamic analogue of the benchmark result in Proposition 1, presents conditions under which this holds.

**Proposition 2** *Consider economies subject to the frictions listed in cases I through IV above. Then, dispersed information is irrelevant if*

1. *there are no pricing complementarities, i.e.  $\theta = \frac{1}{\psi}$ , and*
2. *idiosyncratic shocks are permanent, i.e.  $\rho_b = \rho_z = 1$ .*

To see the intuition behind these conditions, we use (1) and (8) to rewrite revenues and costs:

$$\begin{aligned} \text{Total Revenue}_t &= \Phi_1 P_{it}^{1-\theta} (P_t^\theta B_{it} C_t) = \Phi_1 P_{it}^{-\theta} (P_t^{\theta-\frac{1}{\psi}} B_{it} M_t) \\ \text{Total Cost}_t &= \Phi_2 P_{it}^{-\theta\delta} (P_t^\theta B_{it} C_t)^\delta W_{it} = \Phi_2 P_{it}^{-\theta\delta} (P_t^{\theta-\frac{1}{\psi}} B_{it} M_t)^\delta (M_t Z_{it}) N_{it} . \end{aligned}$$

The aggregate price level  $P_t$  affects revenues and costs through demand and wages as reflected in the two terms  $P_t^{\theta-\frac{1}{\psi}} B_{it} M_t$  and  $M_t Z_{it}$ . The first term shows that aggregate prices enter demand for a firm's product through a relative price effect as well as an aggregate demand effect. When  $\theta = \frac{1}{\psi}$ , these two effects exactly cancel each other, so aggregate prices have no net influence on a firm's current or future demand. Thus, under this parametric restriction, profits are functions solely of the firm's price  $P_{it}$  and particular combinations of exogenous shocks - specifically,  $B_{it} M_t$  and  $M_t Z_{it}$ .

The second condition - on persistence of shocks - ensures that the market signals are sufficient to forecast these combinations. When shocks are equally persistent (recall that the monetary shock is permanent), the most recent realizations of  $B_{it} M_t$  and  $M_t Z_{it}$  contain all the relevant information for characterizing the conditional distributions of  $B_{it+s} M_{t+s}$  and  $M_{t+s} Z_{it+s}$ . Therefore, the additional information available to firms in the full information economy (i.e. the realizations of the underlying shocks  $M_t, B_{it}$  and  $Z_{it}$ ) is irrelevant for the purposes of the price-setting decision.

Finally, from (6), the growth rate of the discount factor bears a one-to-one relationship to the growth rate of money supply. Since  $M_t$  is a random walk, this growth rate is iid. This implies that the joint distribution of  $\left\{\frac{\lambda_{t+s}}{\lambda_t}\right\}$  and the relevant processes for costs and revenues  $\{B_{it+s}M_{t+s}, M_{t+s}Z_{it+s}\}$  are identical under both informational assumptions.

To summarize, in this section, we have established two theoretical benchmarks in assessing the role of dispersed information in a market economy. First, in the absence of information and adjustment lags, markets play a very effective role in the transmission of information. Second, lags or frictions induce departures from this benchmark only to the extent there are strategic interactions in decisions or differences in the dynamic properties of underlying shock processes.

We now turn to studying these departures in the two most commonly used models of nominal frictions - Calvo pricing (case III above) and menu costs (case IV). In the next two sections, we examine, using a combination of analytical and numerical results, the extent to which the combination of dispersed information, strategic complementarities and differences in persistence can generate substantial real effects from nominal shocks.

### 3 Calvo pricing with information frictions

For analytical tractability, we work with a log-quadratic approximation of the firm's profit function. Using  $\hat{x}_t$  to denote the deviation of the variable  $x_t$  from its steady state value, we can write profits as (see Appendix for details):

$$(1 - \theta + \theta\delta) \sum_{t=0}^{\infty} \beta^t \left[ \mathcal{P}_{it+s}^* \hat{p}_{it+s} - \frac{1}{2} \hat{p}_{it+s}^2 \right] + \text{Terms independent of } \{\hat{p}_{it+s}\} \quad , \quad (12)$$

where  $\mathcal{P}_{it+s}^*$  denotes the (log of the) static optimum, i.e. the optimal price under perfect information and fully flexible prices. It is a linear combination of aggregate variables (money supply,  $\hat{m}_{t+s}$  and the aggregate price level,  $\hat{p}_{t+s}$ ) as well as the two idiosyncratic shocks ( $\hat{b}_{it+s}$  and  $\hat{z}_{it+s}$ ),

$$\mathcal{P}_{it+s}^* \equiv (1 - r) \hat{m}_{t+s} + r \hat{p}_{t+s} + \left( \frac{\delta - 1}{1 - \theta + \theta\delta} \right) \hat{b}_{it+s} + \left( \frac{1}{1 - \theta + \theta\delta} \right) \hat{z}_{it+s} \quad .$$

The parameter  $r$  summarizes the effect of the aggregate price on an individual firm's target and thus indexes the degree of strategic complementarity in pricing decisions.

$$r \equiv \frac{\delta - 1}{1 - \theta + \theta\delta} \left( \theta - \frac{1}{\psi} \right) .$$



In every reset period, the firm solves:

$$\max_{\hat{p}_{it}} \sum_{T=1}^{\infty} (1-\xi)^{T-1} \xi \sum_{s=0}^{T-1} \beta^s \mathbb{E}_{it} \left[ \mathcal{P}_{it+s}^* \hat{p}_{it} - \frac{1}{2} \hat{p}_{it}^2 \right].$$

The optimal reset price is

$$\hat{p}_{it}^* = (1 - \beta + \beta\xi) \sum_{s=0}^{\infty} (1 - \xi)^s \beta^s \mathbb{E}_{it} [\mathcal{P}_{it+s}^*] . \quad (13)$$

**Information:** As before, under full information, the firm observes the realization of all aggregate and idiosyncratic shocks, whereas under dispersed information the firm's information set consists only of a history of its revenues and wages. Again, from (1) and (8), we see that this is informationally equivalent to observing a history of the following two signals:

$$\begin{aligned} s_{it}^1 &= \frac{1}{\psi} \hat{m}_t + \left( \theta - \frac{1}{\psi} \right) \hat{p}_t + \hat{b}_{it} \\ s_{it}^2 &= \hat{m}_t + \hat{z}_{it} . \end{aligned} \quad (14)$$

Note that a contemporaneous observation of these two signals allows the firm to infer its static optimum perfectly, since

$$\mathcal{P}_{it}^* = \left( \frac{\delta - 1}{1 - \theta + \theta\delta} \right) s_{it}^1 + \left( \frac{1}{1 - \theta + \theta\delta} \right) s_{it}^2 . \quad (15)$$

We are interested in the response of this economy to innovations in money supply under different informational assumptions. For tractability, we will focus on a limiting case where the size of the nominal shocks becomes arbitrarily small, i.e. as  $\sigma_u^2 \rightarrow 0$ . Since estimates for aggregate nominal disturbances are at least an order of magnitude smaller than idiosyncratic demand and cost shocks, this approximation is empirically plausible.<sup>17</sup>

We analyze the effect of a permanent nominal shock in period  $t$ , i.e.  $u_{t+s} = \Delta > 0$  for  $s = 0, 1, 2, \dots$ . This change in money supply enters the two signals of the firm according to (14). However, since the likelihood of such shocks is arbitrarily small (by assumption), each firm attributes all changes in its signals to the idiosyncratic components  $\hat{b}_{it}$  and  $\hat{z}_{it}$ . For expositional simplicity, we will also assume that both the idiosyncratic shocks are equally persistent, i.e.  $\rho_b = \rho_z = \rho$ .

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<sup>17</sup>In Section 5, we discuss the implications of relaxing this assumption.

We guess (and verify) that the expected aggregate price level in periods following the shock, under both dispersed and full information, can be represented in the form

$$\hat{p}_{t+s} = \tau_1 \Delta + \tau_2 \hat{p}_{t+s-1} , \quad (16)$$

where the coefficients  $\tau_1$  and  $\tau_2$  will vary with the information structure. We can thus compare the price adjustment under different informational assumptions by comparing these coefficients. After  $s$  periods of the shock, the aggregate price level then adjusts to

$$\hat{p}_{t+s} = \tau_1 (1 + \tau_2 \dots + \tau_2^{s-1}) \Delta . \quad (17)$$

Under dispersed information, the firm's time- $t$  expectation of its future target is

$$\mathbb{E}_{it} [\mathcal{P}_{it+s}^*] = \mathbb{E}_{it} \left[ (1-r) \hat{m}_{t+s} + r \hat{p}_{t+s} + \left( \frac{\delta-1}{1-\theta+\theta\delta} \right) \hat{b}_{it+s} + \left( \frac{1}{1-\theta+\theta\delta} \right) \hat{z}_{it+s} \right] .$$

Now, using the fact that firms attribute all shocks to idiosyncratic factors, so that  $\mathbb{E}_{it} m_{t+s} = \mathbb{E}_{it} p_{t+s} = 0$  and  $\mathbb{E}_{it} \hat{b}_{it+s} = \rho^s s_{it}^1$  and  $\mathbb{E}_{it} \hat{z}_{it+s} = \rho^s s_{it}^2$ , we obtain

$$\mathbb{E}_{it} [\mathcal{P}_{it+s}^*] = \left( \frac{\delta-1}{1-\theta+\theta\delta} \right) \rho^s s_{it}^1 + \left( \frac{1}{1-\theta+\theta\delta} \right) \rho^s s_{it}^2 = \rho^s \mathcal{P}_{it}^* .$$

Substituting into (13) and simplifying, we obtain the following expression for the optimal reset price under dispersed information:

$$\hat{p}_{it}^* = \left[ \frac{1-\beta+\beta\xi}{1-\beta\rho(1-\xi)} \right] \mathcal{P}_{it}^* . \quad (18)$$

Then, a few lines of algebra yield  $(\tau_1^{Disp}, \tau_2^{Disp})$ , collected in following result.

**Proposition 3** *As aggregate shocks become arbitrarily small, i.e.  $\sigma_u^2 \rightarrow 0$ , the expected aggregate price level in period  $t+s$  in the dispersed information economy evolves according to (16) with*

$$\tau_1^{Disp} = \xi \frac{(1-r)}{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta} - \xi r} \quad \tau_2^{Disp} = (1-\xi) \frac{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta}}{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta} - \xi r} .$$

It is straightforward to verify that both  $\tau_1^{Disp}$  and  $\tau_2^{Disp}$  are increasing in  $\rho$ . Then, equation (17) implies that the aggregate price adjustment under dispersed information is also increasing in  $\rho$ . This is intuitive - when aggregate shocks are small, firms attribute them almost entirely to idiosyncratic

factors and therefore, expect them to decay at the rate  $\rho$ . When  $\rho < 1$ , this misattribution leads firms to adjust their prices by less than they would under full information. Therefore, aggregate price adjustment is slower and smaller, the larger is the gap between the persistence of idiosyncratic and aggregate shocks.

To derive the comparative statics with respect to  $r$ , it is useful to rewrite the RHS of (17) as the product of two terms - the long-run adjustment  $\tau_1/(1 - \tau_2)$   $\Delta$  and the rate of convergence  $(1 - \tau_2^s)$ . For the long-run adjustment, we observe that

$$\frac{\tau_1^{Disp}}{1 - \tau_2^{Disp}} = \frac{1}{1 + \frac{\beta(1-\xi)}{1-\beta+\beta\xi} \left(\frac{1-\rho}{1-r}\right)} \leq 1. \quad (19)$$

In other words, prices never fully adjust to the nominal shock under dispersed information, unless idiosyncratic shocks are permanent ( $\rho = 1$ ), or prices are completely flexible ( $\xi = 1$ ). The strength of long-run non-neutrality depends on the difference in persistence (a lower  $\rho$ ) and strategic complementarities (higher  $r$ ). In fact, the  $\frac{1-\rho}{1-r}$  term in the denominator shows how complementarities amplify the effects of differences in persistence in the long run.

The expression in equation (19) provides a measure of the extent to which the Hayekian mechanism remains in operation despite the presence of adjustment frictions and complementarities: it shows that aggregate prices can adjust substantially, and in some cases completely even without agents even being aware of the changes in the underlying aggregate conditions ! It also points to the limits of the Hayekian mechanism, by showing how adjustment frictions and differential degrees of persistence can interact to prevent full adjustment, even in the long run. Note, however, that this long-run non-neutrality result relies on the rather stark assumption that the true nature of the shock is never revealed and must therefore be interpreted with caution. As we will see in section 5, it is sensitive to the introduction of additional information about aggregate conditions.<sup>18</sup> The residual adjustment and speed of convergence to neutrality then will be governed by the availability and informativeness of these additional signals.

Complementarities also affect the rate at which prices converge to this long run outcome. From proposition 3, we note that  $\tau_2^{Disp}$  increases with  $r$ , or equivalently,  $(1 - \tau_2^s)$  decreases with  $r$ . In

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<sup>18</sup>It is important to point out that there are two limits in equation (19) and the order is crucial for long-run non-neutrality. More precisely, the term on the left hand side of (19) is  $\lim_{s \rightarrow \infty} \lim_{\sigma_u^2 \rightarrow 0} \frac{d\hat{p}_{t+s}}{du_t}$ . If we reverse the order of the limits, we recover neutrality.

other words, stronger complementarities (higher  $r$ ) limit both the long-run adjustment of prices as well as the speed of convergence.

The mechanism through which complementarities influence price adjustment under dispersed information is novel and distinct from the standard mechanisms. Here, complementarities govern the extent to which aggregate shocks are reflected in the firm-level demand and wage signals. To see this, we substitute (14) into (15) to obtain

$$\mathcal{P}_{it}^* = (1 - r) \hat{m}_t + r \hat{p}_t + \left( \frac{\delta - 1}{1 - \theta + \theta \delta} \right) \hat{b}_{it} + \left( \frac{1}{1 - \theta + \theta \delta} \right) \hat{z}_{it} .$$

The cross-sectional average target price is thus a convex combination of money supply and the aggregate price level,

$$\int \mathcal{P}_{it}^* di = (1 - r) \hat{m}_t + r \hat{p}_t.$$

Thus, a higher  $r$  implies a higher weight of  $\hat{p}_t$ , the aggregate price level (relative to  $\hat{m}_t$ , money supply) in the firms' signals. Since  $\hat{p}_t$  reflects the shock only slowly (while the effect on  $\hat{m}_t$  is immediate), stronger complementarities slow the rate at which the shock enters firm-level information and through that, the rate at which it is incorporated into prices (recall that the optimal reset price is proportional to the target price  $\mathcal{P}_{it}^*$ ).

This mechanism is very different from the one in play both in full information sticky price models and in the dispersed information models of Woodford (2003) or Mankiw and Reis (2002). In those settings, firms receive news that aggregate conditions have changed, but they refrain from fully adjusting to these news because they expect that other firms will not fully adjust either - and hence dampen their responses.<sup>19</sup> In our model, and especially at the small shocks limit, this channel is absent because firms never even consider the possibility of a change in aggregate conditions. Complementarities nevertheless are important through the informational role they play: they determine the speed at which an aggregate shock enters firm-level information, at which point the firms mistakenly attribute these shocks to idiosyncratic factors and respond accordingly. Thus, this is a novel, informational role for complementarities, quite distinct from the strategic effects of

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<sup>19</sup>With full information, the complementarity thus propagates lack of adjustment from firms that have their prices fixed to firms who have the ability to adjust, while in the dispersed information models, the complementarity amplifies an initial lack of adjustment to available information because firms are not exactly certain what other firms have observed, but at its core the two mechanisms are very similar.

complementarities emphasized in the existing literature on nominal price adjustment.

We conclude with two polar cases to isolate the effects of persistence and complementarities. First, without complementarities ( $r = 0$ ), the coefficients in proposition 3 simplify to

$$\tau_1^{Disp} = \xi \frac{1 - \beta(1 - \xi)}{1 - \beta\rho(1 - \xi)} \qquad \tau_2^{Disp} = 1 - \xi .$$

With full information,  $\tau_1^{Full} = \xi$  and  $\tau_2^{Full} = 1 - \xi$ , and therefore  $\tau_1^{Disp} < \tau_1^{Full}$  if and only if  $\rho < 1$ . In other words, without complementarities, informational frictions do not have any effect on the speed of adjustment, but lower the long-term adjustment level when idiosyncratic are relatively less persistent than aggregate shocks. The following result is immediate.

**Claim 1** *Suppose  $r = 0$ , but  $\rho < 1$ . Then, prices in the dispersed information economy respond less to nominal shocks than at the full information benchmark.*

Next, to see the role of complementarities, we consider the case where all shocks are permanent ( $\rho = 1$ ). From Proposition 3 we obtain:

$$\tau_1^{Disp} = \frac{\xi(1 - r)}{1 - \xi r} \qquad \tau_2^{Disp} = \frac{1 - \xi}{1 - \xi r} .$$

Directly, we see that  $\tau_1^{Disp} / (1 - \tau_2^{Disp}) = \tau_1^{Full} / (1 - \tau_2^{Full}) = 1$ . In the Appendix, we show that  $\tau_2^{Disp} > \tau_2^{Full}$ . In other words, with complementarities alone, informational frictions do not affect the long-run level of adjustment, but slow down convergence.

**Claim 2** *Suppose  $\rho = 1$ , but  $r > 0$ . Then, prices in the dispersed information economy respond more slowly to nominal shocks than at the full information benchmark.*

Since overall price adjustment is increasing in  $\rho$  under dispersed information, this result also shows that regardless of parameter values, overall price adjustment is smaller and slower in the dispersed information economy relative to its full information counterpart. Information frictions thus amplify non-neutrality.

**Numerical results:** Finally, we explore the quantitative relevance of dispersed information under reasonable parameter values. We follow a well-worn path from the pricing literature to

parameterize the model. The time period is set to a week and accordingly,  $\beta = 0.999$  targets an annual interest rate of 5 percent. We set the elasticity of substitution  $\theta = 5$ , the curvature of the utility function  $\psi = 2$  and the (inverse of the) returns to scale in production  $\delta = 2$ . Together, they imply complementarity  $r = 0.75$ . This is towards the higher end of the range of micro estimates - see, for example, Burstein and Hellwig (2007) - but slightly lower than the estimates based on aggregate data - see Rotemberg and Woodford (1997). The Calvo parameter  $\xi$  is chosen so that 25 percent of prices are changed every month. The autocorrelation of idiosyncratic shocks,  $\rho$ , is set to match moments of the micro data. Specifically, we choose  $\rho$  and  $\sigma_b^2 = \sigma_z^2 = \sigma^2$  so that the squared deviations of the following simulated moments from their empirical counterparts (which are listed in parentheses) are minimized - average absolute price changes (0.12), standard deviation (in logs) of prices (0.08) and the autocorrelation of log prices (0.67). The first target are in line with estimates reported by a number of studies - Bils and Klenow (2004), Nakamura and Steinsson (2008) and Klenow and Krvystov (2008). The second is derived from statistics reported by Burstein and Hellwig (2007) and the autocorrelation target from Midrigan (2011). This procedure<sup>20</sup> yields a estimate for  $\rho$  of 0.95.

The left panel of Figure 1 shows the impulse response function of (the log of) real balances (i.e.  $\hat{m}_t - \hat{p}_t$ ) for this parameterization (with the shock  $\Delta$  normalized to 1).<sup>21</sup> As the graph shows, dispersed information leads to significantly larger real effects of nominal shocks, especially at longer horizons. For example, under full information, more than 90% of the nominal shock is reflected in aggregate prices within 50 weeks, leading to very modest effects on real output. Under dispersed information, over that same horizon, the price adjustment is only about 20%. The other two panels isolate the effects of differences in persistence and strategic complementarities. The middle panel maintains  $r = 0.75$ , but assumes that the idiosyncratic shocks are permanent while the right panel keeps  $\rho = 0.95$  but sets  $r = 0$ . These two polar cases have similar implications in the short run - real effects are only slightly larger in the dispersed information economy compared to the corresponding full information economy. In the medium to long run, however, the two panels look very different. When all shocks are equally persistent, the amplification from dispersed information

<sup>20</sup>Using additional moments (e.g. percentiles of the price change distribution) does not change our parameter estimates or our results materially.

<sup>21</sup>Since  $y_t = 1/\psi \cdot (m_t - p_t)$ , the impulse response function of real output is proportional to that of real balances.

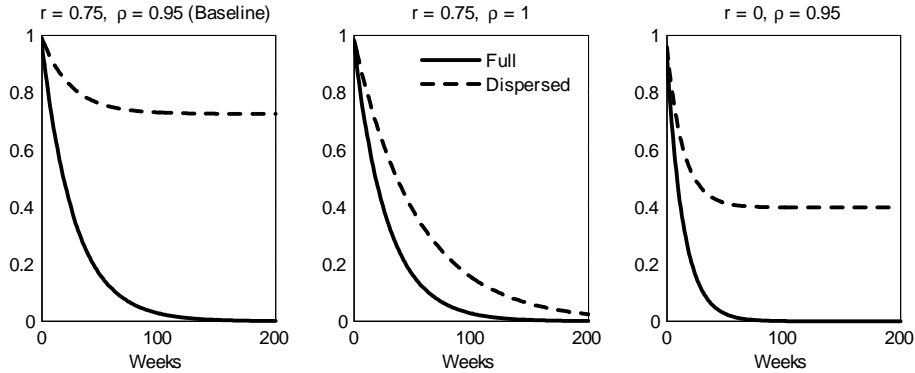


Figure 1: Impulse responses of real balances to a nominal shock under Calvo pricing.

remains modest and decays relatively quickly. The fact that this occurs despite the relatively high degree of complementarity indicates that equilibrium interactions (and the coordination motives they give rise to) do not, by themselves, generate a substantial role for informational frictions. When idiosyncratic shocks are more transitory than aggregate shocks, however, dispersed information leads to a much more prolonged adjustment than under full information.<sup>22</sup> Finally, comparing the left and right panels, we note that strategic complementarities do exert a more significant influence when  $\rho < 1$ . In other words, the interaction of these two elements - complementarities and differences in persistence - can be quite powerful over the medium term.

## 4 Menu costs with information frictions

In this section, we evaluate the role of dispersed information in a calibrated model with menu costs and dispersed information. However, in contrast to the Calvo model studied in the previous section, we work with full-blown non-linear expressions for revenues and costs, rather than a quadratic approximation. This precludes analytical solutions, but allows a more robust quantitative exercise.

We again focus on the limiting case of small aggregate shocks. Specifically, we compute the stationary distribution of prices without aggregate shocks and then subject the economy to a small permanent shock to money supply. The resulting changes in firms' signals are attributed entirely to idiosyncratic factors.

The preference and technology parameters - specifically,  $\beta, \theta, \delta, \psi$  - are set to the same values

<sup>22</sup>Recall that, in this limiting case, prices, even in the long run, do not fully incorporate the shock when  $\rho < 1$ .

	Description	Value
<i>Parameters</i>		
	Time period	1 week
$\beta$	Discount factor	0.996
$\rho$	Autocorrelation of idiosyncratic shocks	0.92
$\sigma$	Standard deviation of idiosyncratic shocks	0.155
$\mathcal{C}$	Menu cost	0.235

Table 1: Calibration summary

as in the previous section. This leaves 3 parameters to be picked - the persistence and variance of the idiosyncratic shocks, i.e.  $(\rho, \sigma^2)$  and the menu cost,  $\mathcal{C}$ .<sup>23</sup> These are chosen to target the same four moments as in the previous section - monthly frequency of price changes (0.25), average absolute price change (0.12), standard deviation of prices (0.08) and monthly autocorrelation of prices (0.67).

Table 1 collects the calibrated parameter values. Note that the estimates for the persistence of idiosyncratic shocks,  $\rho$ , is slightly higher than in the Calvo case studied in the previous section. It is also higher than the corresponding number in the baseline calibration of Golosov and Lucas (2007). This difference arises because we target two new moments, namely autocorrelation and dispersion of prices. Since differences in persistence between aggregate and idiosyncratic shocks are an important source of departures from the Hayekian benchmark, our baseline calibration makes it harder for informational frictions to be relevant.

**Results:** As before, we compare impulse response functions of real balances under full and dispersed information.<sup>24</sup> These are plotted in Figure 2. The real effects from nominal shocks are generally smaller than under Calvo pricing. This is due to the well-known ‘selection effect’ - see Golosov and Lucas (2007). A positive aggregate shock alters the mix of firms adjusting prices towards those increasing prices, which increases the responsiveness of the aggregate price level to the shock. But, in terms of the relevance of informational frictions, the overall pattern is similar to Figure 1. The left panel, which presents our baseline calibration, shows dispersed information

<sup>23</sup>Note that, as before, we are setting  $\rho_b = \rho_z = \rho$  and  $\sigma_b^2 = \sigma_z^2 = \sigma^2$ .

<sup>24</sup>To generate these functions, we simulated 1000 runs of an economy with 10000 firms for 1200 periods, with the realization of the money growth rate for the 1000<sup>th</sup> period fixed at 0.0072. The graphs show the average response of output (normalized by the aggregate shock). We verified numerically that our results are robust to varying these simulation parameters.



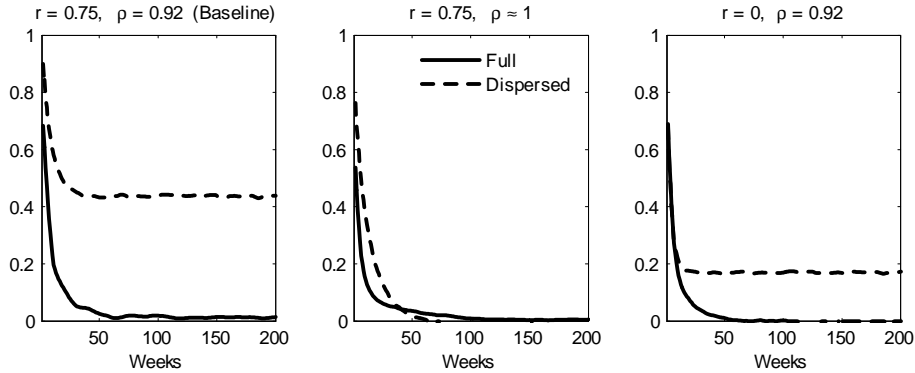


Figure 2: Impulse responses of real balances to a nominal shock with menu costs.

amplifies the effect of the nominal shock in the short run, but the increase is somewhat modest. Over time, however, the gap between the two cases widens and remains persistently high. The middle and right panels turn off differences in persistence and complementarities respectively to isolate their contribution. Again, we see that complementarities by themselves do not induce significant departures from the Hayekian benchmark (the middle panel), but when combined with differences in persistence, they have significant medium term effects (the right panel).

## 5 Discussion

In this section, we examine the robustness of the numerical findings in sections 3 and 4 to variations in information structure, shock processes and parameter values. In the interest of brevity, we will present results of these exercises under either Calvo pricing or menu costs, using the corresponding results from sections 3 or 4 as our baseline.

**Information structure I:** Our analysis so far has assumed that learning, at all horizons, occurs from market-generated signals. This extreme assumption maximizes the potential for dispersed information to be relevant. However, this clearly becomes less and less reasonable at longer horizons. We therefore relax it by introducing direct learning about aggregate shocks. Specifically, we assume that, every period, a fraction  $\phi$  of firms receive a perfectly informative signal of the aggregate shock. This is in the spirit of the sticky information models of Mankiw and Reis (2002). We set  $\phi$  at a relatively small value (0.01) so as to maintain the focus on learning from market signals over short horizons. The impulse response functions with this additional source of learning are

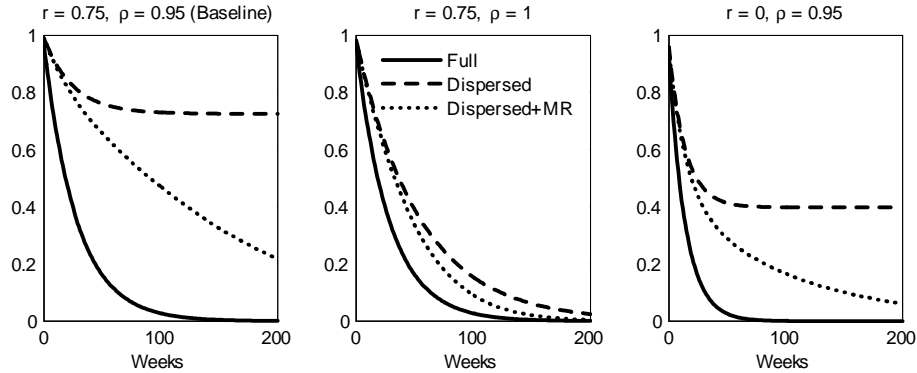


Figure 3: Impulse responses of real balances to a nominal shock under Calvo pricing with additional information.

marked ‘Dispersed + MR’ in Figure 3, which also presents the corresponding lines from Figure 1 (marked ‘Dispersed’). We see that additional information leaves the short run implications essentially unchanged, but over the medium-to-long term, attenuates (and ultimately, eliminates) the gap between full and dispersed information. Note that this only really matters when  $\rho < 1$ . When shocks are equally persistent, as in the center panel, we are relatively close to the Hayekian benchmark and the effect of this additional information is negligible.

**Information structure II:** Our numerical analysis also focused on the limiting case of arbitrarily small aggregate shocks. Relaxing this assumption raises formidable technical challenges. The first is the well-known ‘curse of dimensionality’ which arises in models where the cross-sectional distribution is a relevant state variable. Here, this problem is compounded by the ‘infinite regress’ problem highlighted by Townsend (1983): When actions are strategically linked, firms’ optimal decisions depend on the entire structure higher-order expectations (i.e. their beliefs about others’ beliefs, their beliefs about others’ beliefs about their beliefs...). Thus, the entire structure of higher-order beliefs becomes an state variable. Combined with the non-linearities in the policy functions in the menu cost model, this high dimensionality of the state vector makes aggregation a very challenging task. Finally, the presence of a dynamic filtering problem with endogenous signals makes it difficult to directly apply Kalman filter techniques. In an earlier version of this paper, we overcame these challenges in a simplified version of the menu cost model presented above, by combining the approximation methods in Krusell and Smith (1998) with standard filtering tech-

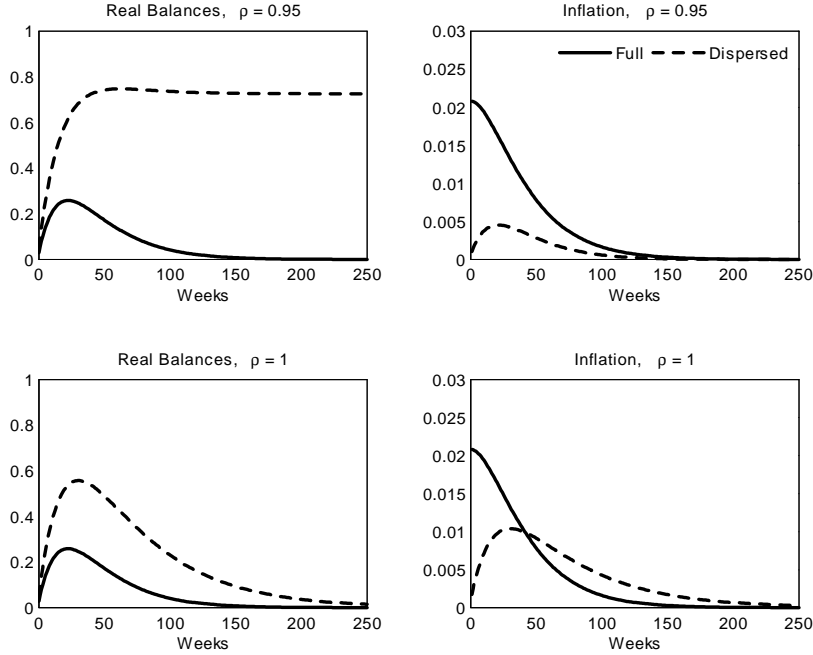


Figure 4: Impulse response to a positive nominal shock with Calvo pricing and persistent shocks to the money growth rate.

niques.<sup>25</sup> The results, especially in the short run, were very close to the limiting case analyzed in this paper. Over a longer horizon, however, the true nature of the shock is ultimately revealed. As with the direct learning above, this has the effect of shrinking (and ultimately, eliminating) the longer term gap between dispersed and full information cases.

**Alternative shock processes (aggregate) :** Next, we introduce serial correlation in innovations to money supply. This specification is commonly used in the literature, partly because it causes the real effect of a nominal shock to peak a few quarters after impact (under the random walk assumption, real effects are highest in the period of impact).<sup>26</sup> Specifically, we assume that

$$\begin{aligned}
 m_t &= m_{t-1} + u_t \\
 u_t &= \rho_u u_{t-1} + v_t \quad v_t \sim N(0, \sigma_v^2).
 \end{aligned}$$

Again, we focus on the limit as  $\sigma_v^2 \rightarrow 0$ . Figure 4 plots the response of output and inflation to a shock  $v_t$ , normalized by the eventual increase in money supply, assuming Calvo pricing.<sup>27</sup> In the

<sup>25</sup>See Hellwig and Venkateswaran (2012).

<sup>26</sup>See discussion in Woodford (2003).

<sup>27</sup>If  $v_t = \Delta$ , the eventual increase in  $m_t = \frac{\Delta}{1-\rho_u}$ . We use a value of  $\rho_u = 0.95$ , which corresponds to a quarterly autocorrelation in money growth of about 0.5, consistent with the estimates in Christiano et al. (2005).

top two panels, we maintain our baseline calibration for the persistence of the idiosyncratic shock,  $\rho = 0.95$ , and plot the impulse response of output and real balances on the left, and of inflation to on the right. In comparison to the random walk case in Figure 1, the overall pattern is similar - dispersed information leads to higher real effects at all horizons but the most striking amplification occurs at longer horizons. In the bottom panels, we set  $\rho = 1$ , making firm-specific shocks fully persistent. Now, prices ultimately fully reflect the shock, though the speed of adjustment is slower than in the full information economy. More interestingly, however, there is still significant short run amplification from the information friction. To see the source of this amplification, note that, under full information, the effects of the current shock on future money growth is correctly anticipated by firms when they reset prices. Under dispersed information, however, firms interpret changes in their signals as arising from (iid) innovations to their idiosyncratic factors. This channel is missing when  $\rho_u = 0$ , i.e. when all shocks were random walks. This explains why the bottom left panel in Figure 4 is markedly different from the corresponding one in Figure 1.

The sluggish response of prices to aggregate shocks contrasts with the (relatively) rapid and permanent adjustment of prices to idiosyncratic shocks. The model with serial correlation in money supply and fully persistent idiosyncratic shocks thus matches the empirical evidence in Boivin et al. (2009) and Maćkowiak et al. (2009) who find that prices reflect sector-specific shocks much more rapidly than aggregate shocks.

**Alternative shock processes (idiosyncratic):** We now consider the implications of an alternative calibration strategy for idiosyncratic shocks in the menu cost model. Midrigan (2011) shows in a full information menu cost model that accounting for the large observed heterogeneity in price changes weakens the selection effect at work in Golosov and Lucas (2007) and therefore, leads to slower price adjustment and larger real effects. We make two modifications to our menu cost model and follow the calibration strategy from Midrigan (2011) to explore how the selection effect interacts with dispersed information. First, in every period, each firm faces zero costs of changing prices with probability  $\phi$ . Second, the two idiosyncratic shocks now evolve according to

the following process

$$\begin{aligned}
 x_{it} &= x_{it-1} && \text{with probability } 1 - \varpi \\
 &= \rho x_{it-1} + u_{it}^x && \text{with probability } \varpi \quad x = \{b, z\}.
 \end{aligned}$$

We parameterize this version of the model as follows. We hold the preference and production parameters at their baseline values. To pin down the idiosyncratic shock and menu cost parameters  $(\rho, \sigma^2, \phi, \varpi, \mathcal{C})$ , we target, as before, (a) an average absolute monthly price change of 12% (b) a frequency of monthly price changes of 25% (c) a standard deviation of prices of 8%, but also the 10th, 25th, 50th, 75th and 90th percentile of price changes<sup>28</sup>. The bottom left panel of Figure 5 plots the impulse response of real balances to a positive nominal shock in this version. The top left panel is taken from Figure 2 and contains the corresponding results from the menu cost model in section 4. We see that, relative to that baseline version, this alternative calibration strategy noticeably increases the relevance of dispersed information, particularly at longer horizons. The intuition rests on the weakening of the selection effect, which occurs exactly for the reasons pointed out by Midrigan (2011). A calibration strategy which matches the heterogeneity in price changes has a much smaller measure of ‘marginal’ firms close to their adjustment thresholds. As a result, the effect of the change in the mix of adjusting firms induced by a nominal shock is also smaller. While this feature tends to raise real effects in both the full and dispersed information economies, it is amplified through an informational channel in the latter. Slower adjustment of aggregate prices implies that firm-level signals reflect the aggregate shock slowly (through the presence of strategic complementarities, see discussion in Section 3). This exacerbates the sluggishness in response of prices and leads to larger, more persistent real effects. To bring this out more clearly, we repeat the exercise without strategic complementarities, i.e. with  $r = 0$  and present results in the right two panels. Now, the relevance of dispersed information under the Midrigan calibration is not that different from what we saw in section 4.

**Parameter choices:** Finally, in Figure 6, we examine the effects of varying the degree of price stickiness,  $\xi$ . The left panel reproduces the baseline version, where the target for monthly price

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<sup>28</sup>The targets are taken from Midrigan (2011) are 0.03, 0.05, 0.09, 0.13 and 0.21. The resulting parameter estimates are  $(\rho, \sigma, \phi, \varpi, \mathcal{C}) = (0.8, 0.315, 0.35, 0.05, 1)$ .

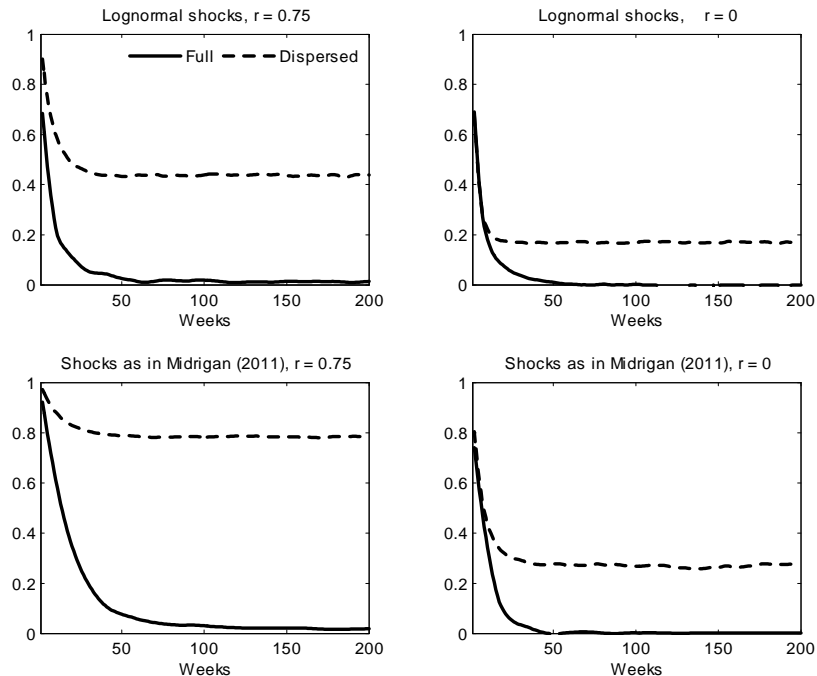


Figure 5: Impulse responses of real balances to a nominal shock in a menu cost model. The top two panels are reproduced from Figure 2.

changes was 25 percent. In the other two panels, that target is changed to 15 and 40 percent. The pattern in all three figures is similar, showing that our main conclusions about the relevance of dispersed information are robust to reasonable changes in the degree of stickiness.

## 6 Conclusion

In this paper we have sought to offer a novel perspective on the respective roles of incomplete information and sticky prices for monetary non-neutrality. At the heart of our analysis is the assumption that firms update and respond to the information that emerges from their market activities. We showed the existence of a "Hayekian benchmark", at which the incorporation of information solely from market activities results in the same outcome as if everyone had full information. This occurs when prices are completely flexible or prices are sticky but there are no pricing complementarities and aggregate and idiosyncratic shocks have the same degree of persistence. Our framework, particularly our modeling of firm-level information as emerging from market activities, further has the advantage that it tightly connects the micro- and macroeconomic properties of price adjustment. By calibrating the key parameters to match micro moments of price adjustment, we were thus able

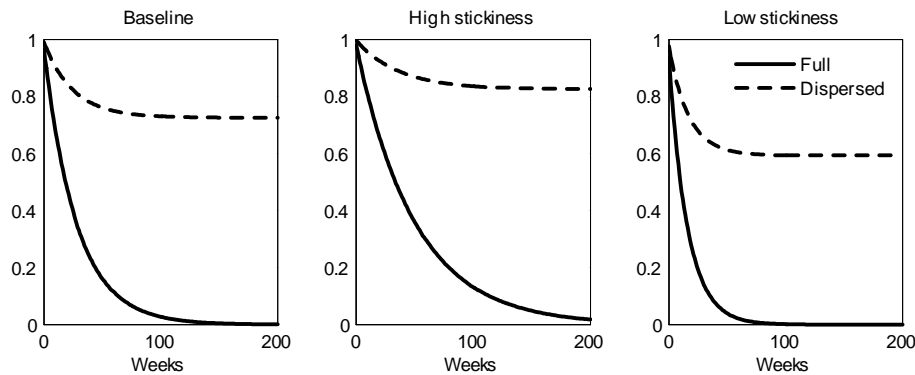


Figure 6: Impact of a positive monetary shock on real balances under Calvo pricing. The baseline panel is reproduced from Figure 1.

to explore quantitatively the potential for complementarities and differential rates of persistence to generate monetary non-neutrality.

The departures from the Hayekian benchmark can also be interpreted through the lens of the beauty contest analogy in Keynes (1936), where he writes : *"Thus, certain classes of investment are governed by the average expectation of those who deal on the stock market, as revealed in the price, rather than by the genuine expectations of the professional entrepreneur."* Keynes draws the distinction between the valuation signals generated in markets and the underlying "fundamental" values (those of the professional entrepreneur) that emerges in a context of uncertainty about economic conditions. Although his analogy focuses on investment, not pricing decisions, the quote implicitly contains the key elements that we have identified as important for departures from the Hayekian benchmark: forward-looking decisions (in this case, investment) which require agents to forecast future market conditions, complementarities that introduce a need to forecast other investors' expectations, and the focus on stock markets in which there tends to be a significant and persistent *common value* component. Our framework thus also allows us to draw a formal connection between the rather contrasting views on the informational role of markets and prices expressed in the writings of Hayek and Keynes, and to identify the implicit assumptions on which these views are based.

There are several avenues for future research. To keep the focus squarely on the firms' price-setting problem, we abstracted, as does most of the New Keynesian literature, from frictions on

households. A natural next step is to examine the strength of the Hayekian mechanism when both households and firms are imperfectly informed. A similar comment applies to augmenting the analysis by adding other shocks, especially ones that interfere with the Hayekian mechanism.<sup>29</sup> Finally, our analysis reveals the crucial role played by forward-looking decisions in generating a role for informational frictions in a market economy. These insights about the interaction of dynamic decisions and imperfect information are also applicable to, and might hence be potentially relevant for understanding, other forward-looking decisions, among other things, investment, savings, or hiring.<sup>30</sup>

## References

- Alvarez, F. E., F. Lippi, and L. Paciello (2011). Optimal price setting with observation and menu costs. *The Quarterly Journal of Economics* 126(4), 1909–1960.
- Amador, M. and P.-O. Weill (2010). Learning from prices: Public communication and welfare. *Journal of Political Economy* 118(5), pp. 866–907.
- Angeletos, G.-M. and J. La’O (2009). Incomplete information, higher-order beliefs and price inertia. *Journal of Monetary Economics* 56, S19–S37.
- Angeletos, G.-M. and J. La’O (2010). Noisy business cycles. In *NBER Macroeconomics Annual 2009, Volume 24*, NBER Chapters, pp. 319–378. National Bureau of Economic Research, Inc.
- Angeletos, G.-M. and J. La’O (2012). Optimal monetary policy with informational frictions. Mimeo, MIT.
- Angeletos, G.-M. and A. Pavan (2007). Efficient use of information and social value of information. *Econometrica* 75(4), 1103–1142.
- Atolia, M. and R. Chahrour (2013). Intersectoral linkages, diverse information, and aggregate dynamics in a neoclassical model. Mimeo, Boston College.

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<sup>29</sup> An obvious example would be mark-up shocks. When the optimal markup over marginal costs is stochastic, firms’ sales and wage signals are no longer sufficient for their optimal price, even in the absence of nominal frictions.

<sup>30</sup> For other work exploring implications of market-based information, see Atolia and Chahrour (2013) for a multi-sector investment model and Venkateswaran (2014) for a labor market application. In ongoing work, we embed this mechanism in a real business cycle setting and study propagation of aggregate productivity shocks.



- Ball, L. and D. Romer (1990). Real rigidities and the non-neutrality of money. *The Review of Economic Studies* 57(2), 183–203.
- Bils, M. and P. J. Klenow (2004). Some evidence on the importance of sticky prices. *Journal of Political Economy* 112(5), 947–985.
- Boivin, J., M. P. Giannoni, and I. Mihov (2009). Sticky prices and monetary policy: Evidence from disaggregated us data. *American Economic Review* 99(1), 350–384.
- Burstein, A. T. and C. Hellwig (2007). Prices and Market Shares in a Menu Cost Model. NBER Working Paper No. 13455.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383 – 398.
- Caplin, A. and J. Leahy (1991). State-dependent pricing and the dynamics of money and output. *The Quarterly Journal of Economics* 106(3), pp. 683–708.
- Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1), 1–45.
- Coibion, O. and Y. Gorodnichenko (2012). What can survey forecasts tell us about information rigidities? *Journal of Political Economy* 120(1), 116–159.
- Eichenbaum, M., N. Jaimovich, and S. Rebelo (2011). Reference prices, costs, and nominal rigidities. *The American Economic Review*, 234–262.
- Galí, J. (2009). *Monetary Policy, inflation, and the Business Cycle: An introduction to the new Keynesian Framework*. Princeton University Press.
- Golosov, M. and R. E. Lucas (2007). Menu costs and phillips curves. *Journal of Political Economy* 115(2), 171–199.
- Gorodnichenko, Y. (2010). Endogenous information, menu costs and inflation persistence. NBER Working Paper No. 14184.

- Graham, L. and S. Wright (2010). Information, heterogeneity and market incompleteness. *Journal of Monetary Economics* 57(2), 164 – 174.
- Grossman, S. J. and J. E. Stiglitz (1980). On the impossibility of informationally efficient markets. *The American Economic Review* 70(3), pp. 393–408.
- Hayek, F. A. (1945). The use of knowledge in society. *The American Economic Review* 35(4), pp. 519–530.
- Hellwig, C. (2005). Heterogeneous information and the welfare effects of public information disclosures. Mimeo, UCLA.
- Hellwig, C. and L. Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies* 76(1), 223–251.
- Hellwig, C. and V. Venkateswaran (2009). Setting the right prices for the wrong reasons. *Journal of Monetary Economics* 56(Supplement 1), S57 – S77.
- Hellwig, C. and V. Venkateswaran (2012). Hayek vs keynes: Dispersed information and market prices in a price-setting model. Mimeo, Penn State University.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. London: McMillan.
- Klenow, P. J. and O. Kryvstov (2008). State-dependent or time-dependent pricing: Does it matter for recent u.s. inflation? *Quarterly Journal of Economics* 123(3), 863–904.
- Lorenzoni, G. (December 2009). A theory of demand shocks. *The American Economic Review* 99, 2050–2084(35).
- Lucas, R. (1972, April). Expectations and the neutrality of money. *Journal of Economic Theory* 4(2), 103–124.
- Maćkowiak, B., E. Moench, and M. Wiederholt (2009). Sectoral price data and models of price setting. *Journal of Monetary Economics* 56, S78–S99.

- Mackowiak, B. A. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review* 99(2), 769–803.
- Mackowiak, B. A. and M. Wiederholt (2010). Business cycle dynamics under rational inattention. Mimeo, European Central Bank/Nothwestern University.
- Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics* 117(4), 1295–1328.
- Melosi, L. (2012). Signaling effects of monetary policy.
- Melosi, L. (2014). Estimating models with dispersed information. *American Economic Journal: Macroeconomics* 6(1), 1–31.
- Midrigan, V. (2011). Menu costs, multiproduct firms, and aggregate fluctuations. *Econometrica* 79(4), 1139–1180.
- Morris, S. and H. S. Shin (2002). Social value of public information. *The American Economic Review* 92, 1521–1534(14).
- Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics* 123(4), 1415–1464.
- Nimark, K. (2008). Dynamic pricing and imperfect common knowledge. *Journal of Monetary Economics* 55(2), 365 – 382.
- Paciello, L. and M. Wiederholt (2014). Exogenous information, endogenous information, and optimal monetary policy. *The Review of Economic Studies* 81(1), 356–388.
- Phelps, E. S. (1970). Introduction: The new microeconomics in employment and inflation theory. In E. e. a. Phelps (Ed.), *Microeconomic Foundations of Employment and Inflation Theory*. New York: Norton.
- Rotemberg, J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. In *NBER Macroeconomics Annual 1997, Volume 12*, pp. 297–361. MIT Press.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88(1), pp. 1–23.

Venkateswaran, V. (2014). Heterogeneous information and labor market fluctuations. Mimeo, NYU Stern.

Woodford, M. D. (2003). Imperfect Common Knowledge and the Effects of Monetary Policy. In P. A. et al. (Ed.), *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*.

## Appendix A Proofs of results

### A.1 Proposition 1:

The firm's optimality condition is given by

$$\underbrace{\Phi_1 P_{it}^{-\theta} \mathbb{E}_{it}[\lambda_t P_t^\theta B_{it} C_t]}_{\text{Exp. Marginal Revenue}} = \underbrace{\Phi_2 P_{it}^{-\theta\delta-1} \mathbb{E}_{it}[\lambda_t (P_t^\theta B_{it} C_t)^\delta W_{it}]}_{\text{Exp. Marginal Cost}}$$

Under dispersed information, the firm observes sales and its wage bill, which are informationally equivalent to observing  $P_t^\theta B_{it} C_t$  and  $W_{it}$ . Then,  $\mathbb{E}_{it}[\lambda_t (P_t^\theta B_{it} C_t)^\delta W_{it}] = (P_t^\theta B_{it} C_t)^\delta W_{it} \mathbb{E}_{it}[\lambda_t]$  and  $\mathbb{E}_{it}[\lambda_t P_t^\theta B_{it} C_t] = P_t^\theta B_{it} C_t \mathbb{E}_{it}[\lambda_t]$ . Substituting into the optimality condition yields the firm's optimality condition under full information,  $\Phi_1 P_{it}^{-\theta} P_t^\theta B_{it} C_t = \Phi_2 P_{it}^{-\theta\delta-1} (P_t^\theta B_{it} C_t)^\delta W_{it}$ . Since the two economies are identical in all other aspects, it follows that the full information equilibrium remains an equilibrium under dispersed information.

### A.2 Proposition 2:

If  $\theta = \frac{1}{\psi}$ , then total revenue and cost in period  $t + s$  become

$$\text{Total Revenue}_t = \Phi_1 P_{it+s}^{-\theta} B_{it+s} M_{t+s} \tag{20}$$

$$\text{Total Cost}_t = \Phi_2 P_{it+s}^{-\theta\delta} (B_{it+s} M_{t+s})^\delta M_{t+s} Z_{it+s} \tag{21}$$

In other words, profits, marginal revenues and costs do not depend on  $P_t$ . We will exploit this property in our discussion of each of the cases below. Since the full information and dispersed information economies only differ in the intermediate producers' information in the pricing problem,

it suffices to show that dispersed information does not affect their optimal prices to show that it is irrelevant for equilibrium allocations.

**Case I:** The first order condition of the firm is

$$\underbrace{\Phi_1 P_{it}^{-\theta} \mathbb{E}_{it-N}[\lambda_t B_{it} M_t]}_{\text{Exp. Marginal Revenue}} = \underbrace{\Phi_2 P_{it}^{-\theta\delta-1} \mathbb{E}_{it-N}[\lambda_t (B_{it} M_t)^\delta M_t Z_{it}]}_{\text{Exp. Marginal Cost}} \quad (22)$$

Since  $\lambda_t = \hat{v}\beta^t/M_t$ , the optimal price given by

$$P_{it}^{1-\theta+\theta\delta} = K \cdot \frac{\mathbb{E}_{it-N}[(B_{it} M_t)^\delta Z_{it}]}{\mathbb{E}_{it-N}[B_{it}]} = K \cdot \frac{\mathbb{E}_{it-N}(e^{\delta b_{it} + \delta m_t + z_{it}})}{\mathbb{E}_{it-N}(e^{b_{it}})}$$

For some constant  $K > 0$ . If shocks are permanent, i.e.  $\rho_b = \rho_z = 1$ , this becomes

$$P_{it}^{1-\theta+\theta\delta} = K \cdot \frac{\mathbb{E}_{it-N} e^{\delta(b_{it-N} + m_{t-N} + z_{it-N})} e^{\tilde{V}_{it}}}{\mathbb{E}_{it-N} e^{b_{it-N} + m_{t-N}} e^{\tilde{U}_{it}}}$$

where  $\tilde{U}_{it}$  and  $\tilde{V}_{it}$  are functions of  $\{u_{t-s}, u_{it-s}, v_{it-s}\}_{s=0}^{N-1}$ . Under dispersed information, the signals in period  $t - N$  allow the firm to perfectly infer the combinations  $b_{it-N} + m_{t-N}$  and  $z_{it-N} + m_{t-N}$ . Obviously, these combinations are known to the firm in period  $t - N$  under full information. Then, under both informational assumptions, the above expression can be written as

$$P_{it}^{1-\theta+\theta\delta} = K \cdot \frac{e^{\delta(b_{it-N} + m_{t-N}) + z_{it-N}} \mathbb{E}_{it-N} e^{-m_{t-N}} e^{\tilde{V}_{it}}}{e^{\rho^N(b_{it-N} + m_{t-N})} \mathbb{E}_{it-N} e^{-m_{t-N}} e^{\tilde{U}_{it}}} = K \cdot \frac{e^{\delta(b_{it-N} + m_{t-N}) + (m_{t-N} + z_{it-N})} \mathbb{E}_{it-N} e^{\tilde{V}_{it}}}{e^{(b_{it-N} + m_{t-N})} \mathbb{E}_{it-N} e^{\tilde{U}_{it}}}$$

But  $\mathbb{E}_{it-N} e^{\tilde{V}_{it}}$  and  $\mathbb{E}_{it-N} e^{\tilde{U}_{it}}$  are the same under both full and dispersed information. Therefore, it follows that optimal prices will be identical in both economies.

**Case II:** When a price is set for  $N$  periods at a time, the optimal price is characterized by

$$P_{it}^{1-\theta+\theta\delta} = K \cdot \frac{\sum_{s=0}^N \mathbb{E}_{it} \beta^s [(B_{it+s} M_{t+s})^\delta M_{t+s} Z_{it+s}]}{\sum_{s=0}^N \beta^s \mathbb{E}_{it} [B_{it+s} M_{t+s}]}$$

for some  $K > 0$ . When all shocks are permanent, we rewrite the numerator and denominator as

$$\begin{aligned} \sum_{s=0}^N \beta^s \mathbb{E}_{it} [(B_{it+s} M_{t+s})^\delta M_{t+s} Z_{it+s}] &= \sum_{s=0}^N \beta^s \mathbb{E}_{it} e^{\delta(b_{it} + m_t) + m_t + z_{it}} e^{\hat{V}_{it+s}} = e^{\delta(b_{it} + m_t) + m_t + z_{it}} \sum_{s=0}^N \beta^s \mathbb{E}_{it} e^{\hat{V}_{it+s}} \\ \sum_{s=0}^N \beta^s \mathbb{E}_{it} [B_{it+s} M_{t+s}] &= \sum_{s=0}^N \beta^s \mathbb{E}_{it} e^{(b_{it} + m_t)} e^{\hat{U}_{it+s}} = e^{(b_{it} + m_t)} \sum_{s=0}^N \beta^s \mathbb{E}_{it} e^{\hat{U}_{it+s}} \end{aligned}$$

where the second equality uses the fact that  $b_{it} + m_t$  and  $m_t + z_{it}$  are in the firm's information set under both full information and dispersed information. The random variables  $\hat{U}_{it+s}$  and  $\hat{V}_{it+s}$  only

depend on future realizations of shocks, which are orthogonal to information available at time  $t$  in both cases. Thus, the firm's optimal pricing decision is not affected by dispersed information.

**Case III:** With Calvo pricing, the optimal reset price is given by

$$P_{it}^{1-\theta+\theta\delta} = K \cdot \frac{\sum_{T=1}^{\infty} (1-\xi)^{T-1}\xi \left( \sum_{s=0}^{T-1} \beta^s \mathbb{E}_{it}[(B_{it+s}M_{t+s})^\delta M_{t+s}Z_{it+s}] \right)}{\sum_{T=1}^{\infty} (1-\xi)^{T-1}\xi \left( \sum_{s=0}^{T-1} \beta^s \mathbb{E}_{it}[B_{it+s}M_{t+s}] \right)}$$

where  $\xi$  is the (exogenous) probability of resetting prices in any given period. It is easy to see that the logic of the proof for Case II goes through exactly - when all shocks are permanent, both the numerator and denominator are identical under full and dispersed information.

**Case IV:** We establish the irrelevance result in 2 steps. First, we solve the following Bellman equation under full and dispersed information and show that the solutions (and the induced policy functions) coincide:  $\tilde{V}(P_{it-1}, \mathcal{I}_{it}^{Full}) = \tilde{V}(P_{it-1}, \mathcal{I}_{it}^{Disp})$ , where

$$\begin{aligned} \tilde{V}(P_{it-1}, \mathcal{I}_{it}) = \max \quad & \{ \mathbb{E}_{it}[\Pi(P_{it-1}, M_t, B_{it}, Z_{it}) + \beta \frac{\lambda_{t+1}}{\lambda_t} \tilde{V}(P_{it-1}, \mathcal{I}_{it+1})], \\ & \max_P \mathbb{E}_{it}[\Pi(P, M_t, B_{it}, Z_{it}) - W_{it}\mathcal{C} + \beta \frac{\lambda_{t+1}}{\lambda_t} \tilde{V}(P, \mathcal{I}_{it+1})] \}. \end{aligned} \quad (23)$$

Second, we prove that, when all shocks are permanent, the policy functions implied by this Bellman equation correspond to the ones for (11) in the text, under both informational assumptions.

Let  $V^*$  be the solution to the Bellman equation (23) under dispersed information i.e. under  $\mathcal{I}_{it}^{Disp} \equiv \{B_{it-s}M_{t-s}, Z_{it-s}M_{t-s}\}_{s=0}^{\infty}$ . We need to show that  $V^*$  also solves the functional equation (11) under  $\mathcal{I}_{it}^{Full} \equiv \{B_{it-s}, M_{t-s}, Z_{it-s}\}_{s=0}^{\infty}$ . We begin by conjecturing that continuation values are the same under both informational assumptions:

$$\mathbb{E} \left[ \frac{\lambda_{t+1}}{\lambda_t} V^*(P, \mathcal{I}_{it+1}) \mid \mathcal{I}_{it}^{Full} \right] = \mathbb{E} \left[ \frac{\lambda_{t+1}}{\lambda_t} V^*(P, \mathcal{I}_{it+1}) \mid \mathcal{I}_{it}^{Disp} \right] \quad (24)$$

First, recall that both revenues and costs are functions of exactly the combinations of the current realizations of the shocks contained in  $\mathcal{I}_{it}^{Disp}$ . Then,  $\mathcal{I}_{it}^{Disp}$  contains the sufficient statistics for forecasting current profits, i.e.  $\mathbb{E}[\Pi(P, \cdot) \mid \mathcal{I}_{it}^{Full}] = \mathbb{E}[\Pi(P, \cdot) \mid \mathcal{I}_{it}^{Disp}]$ . Therefore, given the conjecture (24) about continuation values, the value of holding prices unchanged is the same under full and dispersed information. The same holds for the value of changing prices. Therefore, since both parts inside the max operator on the right hand side of (23) are the same, it follows that the maximized

value is the same. Thus,  $\tilde{V}(P_{it}, \mathcal{I}_{it}^{Full}) = V^*(P_{it}, \mathcal{I}_{it}^{Disp})$ . Now, if the  $t-1$  expectation of this expression, weighted by  $\frac{\lambda_t}{\lambda_{t-1}}$  is the same under the two informational assumptions, we have verified the initial guess about continuation values, establishing that  $V^*$  is a fixed point for the full information problem as well.

To do this, we note that  $V^*(P_{it}, \mathcal{I}_{it}^{Disp})$  is a function of the price  $P_{it}$  and the two sufficient statistics  $B_{it}M_t$  and  $Z_{it}M_t$ , while the growth rate of the discount factor is a function solely of the money growth rate  $\frac{M_t}{M_{t-1}}$ , an iid random variable. When all shocks are permanent, the  $t-1$  realizations,  $B_{it-1}M_{t-1}$  and  $Z_{it-1}M_{t-1}$ , are sufficient for characterizing the one-period ahead conditional joint distribution of the three random variables  $\left(B_{it}M_t, Z_{it}M_t, \frac{M_t}{M_{t-1}}\right)$ . It then follows that the conditional expectation of  $\frac{\lambda_t}{\lambda_{t-1}}V^*(P_{it}, \mathcal{I}_{it}^H)$  in period  $t-1$  under dispersed information must coincide with that under full information. Thus, we have shown that  $V^*$  also solves the functional equation (23) under full information, implying that prices and quantities are also identical under both informational assumptions.

Finally, we need to show that the policy functions of  $V^*$  coincide with those of (11). This is easy to see under full information. To see that this is also the case under dispersed information, note that the full information optimal policy can be expressed purely in terms of the elements in  $\mathcal{I}_{it}^{Disp}$ , which implies that it will continue to be optimal if we multiply both sides of (23) by  $\lambda_t$  and take expectations conditional on  $\mathcal{I}_{it}^{Disp}$ . This yields the original value function  $V$  in (11), completing our proof of the irrelevance of dispersed information.

### A.3 Firm's objective under Calvo pricing

We begin by deriving a log-quadratic approximation of the firm's discounted profits:

$$(1 - \theta + \theta\delta) \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \left[ \left( \frac{\delta - 1}{1 - \theta + \theta\delta} \right) (\hat{y}_{t+s} + \hat{b}_{it+s} + \theta\hat{p}_{t+s}) + \left( \frac{1}{1 - \theta + \theta\delta} \right) \hat{w}_{it+s} \right] \hat{p}_{it+s} - \frac{1}{2} \hat{p}_{it+s}^2 \right\}$$

where  $\hat{x}$  denotes the log-deviation of the variable  $x$  from its steady state value. Using the equilibrium conditions from the household's problem, we substitute  $\hat{y}_{t+s} = (\hat{m}_{t+s} - \hat{p}_{t+s})/\psi$  and  $\hat{w}_{it+s} = \hat{m}_{t+s} + \hat{z}_{it+s}$  into the expression for profits to obtain the expression in the text

$$(1 - \theta + \theta\delta) \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \mathcal{P}_{it+s}^* \hat{p}_{it+s} - \frac{1}{2} \hat{p}_{it+s}^2 \right\}$$

where

$$\begin{aligned}\mathcal{P}_{it+s}^* &\equiv (1-r)\hat{m}_{t+s} + r\hat{p}_{t+s} + \left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\hat{b}_{it+s} + \left(\frac{1}{1-\theta+\theta\delta}\right)\hat{z}_{it+s} \\ r &= \frac{\delta-1}{1-\theta+\theta\delta}\left(\theta - \frac{1}{\psi}\right)\end{aligned}$$

In every reset period, the firm solves

$$\max_{\hat{p}_{it}} \sum_{T=1}^{\infty} (1-\xi)^{T-1}\xi \sum_{s=0}^{T-1} \beta^s \left( \mathbb{E}_{it} [\mathcal{P}_{it+s}^*] \hat{p}_{it} - \frac{1}{2}\hat{p}_{it}^2 \right)$$

#### A.4 Proof of proposition 3

From (10), the average rest price in period  $t$  is

$$\int \hat{p}_{it}^* di = (1-\beta+\beta\xi) \sum_{s=0}^{\infty} (1-\xi)^s \beta^s \int \mathbb{E}_{it} [\mathcal{P}_{it+s}^*] di.$$

As aggregate shocks become arbitrarily small, firms attribute all variations in their signals to the idiosyncratic shocks, i.e.  $\mathbb{E}_{it}(\hat{b}_{it}|s_{it}^1, s_{it}^2) = s_{it}^1$  and  $\mathbb{E}_{it}(\hat{z}_{it}|s_{it}^1, s_{it}^2) = s_{it}^2$ . Then, the expectation of the future target  $\mathcal{P}_{it+s}^*$  is

$$\begin{aligned}\mathbb{E}_{it} [\mathcal{P}_{it+s}^*] &= \left(\frac{\delta-1}{1-\theta+\theta\delta}\right) \rho^s \mathbb{E}_{it} \hat{b}_{it} + \left(\frac{1}{1-\theta+\theta\delta}\right) \rho^s \mathbb{E}_{it} \hat{z}_{it} \\ &= \left(\frac{\delta-1}{1-\theta+\theta\delta}\right) \rho^s s_{it}^1 + \left(\frac{1}{1-\theta+\theta\delta}\right) \rho^s s_{it}^2\end{aligned}$$

Integrating across resetting firms, we have  $\int \mathbb{E}_{it}(\hat{b}_{it+s}) di = \rho^s s_{it}^1 = \rho^s \left[ \frac{1}{\psi} \hat{m}_t + \left(\theta - \frac{1}{\psi}\right) \hat{p}_t \right]$  and  $\int \mathbb{E}_{it}(\hat{z}_{it+s}) di = \rho^s \Delta_{t+s}$ . Substituting these integrals into the condition for the average reset price, we obtain

$$\begin{aligned}\int \hat{p}_{it}^* di &= (1-\beta+\beta\xi) \sum_{s=0}^{\infty} (1-\xi)^s \beta^s \rho^s \left\{ \left(\frac{\delta-1}{1-\theta+\theta\delta}\right) \left[ \frac{1}{\psi} \hat{m}_t + \left(\theta - \frac{1}{\psi}\right) \hat{p}_t \right] + \frac{\hat{m}_t}{1-\theta+\theta\delta} \right\} \\ &= \frac{1-\beta+\beta\xi}{1-(1-\xi)\beta\rho} [(1-r)\hat{m}_t + r\hat{p}_t]\end{aligned}$$

Substituting into the law of motion for the aggregate price,

$$\hat{p}_t = \xi \int \hat{p}_{it}^* di + (1-\xi)\hat{p}_{t-1} = \xi \frac{1-\beta+\beta\xi}{1-(1-\xi)\beta\rho} [(1-r)\Delta + r\hat{p}_t] + (1-\xi)\hat{p}_{t-1}$$

Collecting terms, we arrive at the coefficients in Proposition 3.

**Proof of Claim 1:** This follows directly from the expressions for  $\tau_1^{Full}$  and  $\tau_1^{Disp}$ .



**Proof of Claim 2:** By a standard guess-and-verify approach (see, for example, Galí (2009)), the response coefficients in the full information economy satisfy

$$\tau_1^{Full} = \frac{\sqrt{[1 - \beta + \chi(1 - r)]^2 + 4\beta\chi(1 - r)} - (1 - \beta + \chi(1 - r))}{2\beta}$$

where  $\chi = \frac{\xi}{1 - \xi}(1 - \beta + \beta\xi)$ , and  $\tau_2^{Full} = 1 - \tau_1^{Full}$ . Now, it's easy to check that

$$\tau_1^{Full} > \frac{\sqrt{4\beta\chi(1 - r)}}{2\beta} = \sqrt{\frac{\xi}{1 - \xi}(1 - \beta + \beta\xi) \frac{(1 - r)}{\beta}} > \sqrt{\frac{\xi^2(1 - r)}{1 - \xi}}$$

Then,

$$\begin{aligned} \left(\tau_1^{Full}\right)^2 - \left(\tau_1^{Disp}\right)^2 &> \xi^2 \left(\frac{1 - r}{1 - \xi}\right) - \xi^2 \left(\frac{1 - r}{1 - \xi r}\right)^2 = \frac{\xi^2(1 - r)(r^2\xi^2 - 3r\xi + r + \xi)}{(1 - \xi)(1 - \xi r)^2} \\ &= \frac{\xi^2(1 - r)}{(1 - \xi)(1 - \xi r)^2} \left[ \xi^2(1 - r)^2 + r(1 - \xi)^2 + (1 - r)\xi(1 - \xi) \right] > 0 \end{aligned}$$

This implies  $\tau_2^{Disp} = 1 - \tau_1^{Disp} > 1 - \tau_1^{Full} = \tau_2^{Full}$ .