Partial Equilibrium Thinking, Extrapolation, and Bubbles*

Francesca Bastianello[†] Paul Fontanier[‡]

December 15, 2023

Abstract

We develop a dynamic theory of "Partial Equilibrium Thinking" (PET), which micro-founds time-varying price extrapolation. Extrapolative beliefs are present at all times, but only sometimes manifest themselves in explosive ways. Consistent with the Kindleberger (1978) narrative of bubbles, we distinguish between normal times shocks and "displacement shocks." In normal times, PET generates constant extrapolation and momentum. By contrast, following a displacement shock that increases uncertainty, PET leads to stronger and time-varying extrapolation, triggering bubbles and endogenous crashes. Our theory sheds light on both normal times market dynamics and the Kindleberger (1978) narrative of bubbles within a unified framework.

Keywords: Bubbles and crashes, beliefs, misinference, partial equilibrium thinking, extrapolation, micro-foundations.

*Previous versions have been circulated under the title "Partial Equilibrium Thinking in Motion." We are extremely grateful to John Campbell, Sam Hanson, Andrei Shleifer, Joshua Schwartzstein, Jeremy Stein, and Adi Sunderam for their invaluable support and guidance. We thank Malcom Baker, Nicholas Barberis, Gabriel Chodorow-Reich, Kent Daniel, Emmanuel Farhi, Xavier Gabaix, Nicola Gennaioli, Robin Greenwood, Oliver Hart, Spencer Kwon, David Laibson, Shengwu Li, Ian Martin, Peter Maxted, Stefan Nagel, Cameron Peng, Matthew Rabin, Elisa Rubbo, Giorgio Saponaro, Ludwig Straub, Tomasz Strzalecki, Johnny Tang, Boris Vallée, Dimitri Vayanos, Luis Viceira, Jessica Wachter, Lingxuan Wu; as well as participants at Harvard Lunch Workshops (Finance, Macro, Theory, Behavioral, and Contracts), the 10th Miami Behavioral Finance Conference, the LBS Summer Symposium, and the Virtual Israel Macro Meeting; and seminars at LSE, UCL, Imperial College Business School, Oxford Saïd, Columbia GSB, Kellogg, Yale SOM, Chicago Booth, HBS, HEC Paris, Stanford GSB, UCLA Anderson and Princeton University for their thoughtful comments and discussions. Francesca Bastianello is grateful for support from the Alfred P. Sloan Foundation Pre-doctoral Fellowship in Behavioral Macroeconomics, awarded through the NBER.

†Chicago Booth: francesca.bastianello@chicagobooth.edu

‡Yale School of Management: paul.fontanier@yale.edu

Electronic copy available at: https://ssrn.com/abstract=4666338

Sustained periods of over-optimism that eventually end in a crash are at the heart of many macro-economic events, such as stock market bubbles, house price bubbles, investment booms, credit cycles, or financial crises (Bagehot 1873, Galbraith 1954, Kindleberger 1978, Shiller 2000, Jordà et al. 2015, Greenwood et al. 2022). Given the real consequences of bubbles and crashes, there has been widespread interest in understanding their anatomy and the beliefs that support them.

In terms of anatomy, Kindleberger (1978)'s historical narrative of bubbles provides us with some guidance, by identifying three key stages of bubbles and crashes. The first stage is characterized by what Kindleberger refers to as a *displacement*, "some outside event that changes horizons, expectations, anticipated profit opportunities, behavior." Examples include technological revolutions, such as the railroads in the 1840s, the radio and automobiles in the 1920s, and the internet in the 1990s, or financial innovations such as securitization prior to the 2008 financial crisis. The second stage is characterized by *euphoria* and acceleration. As investors respond to such shocks, the good news leads to a wave of optimism and rising prices. This in turn encourages further buying in a self-sustaining feedback between prices and beliefs that decouples prices from fundamentals. More recent empirical evidence has also shown that this stage is also associated with destabilizing speculation (De Long et al. 1990, Brunnermeier and Nagel 2004), accelerating and convex price paths (Greenwood et al. 2019), and heavy trading (Hong and Stein 2007, Barberis 2018, DeFusco et al. 2020). Eventually, in the third stage, agents who rode the bubble exit, leading to a *crash*.

Turning to beliefs, early theories of bubbles maintain the assumption of rational expectations (Tirole 1985, DeMarzo et al. 2007). However, as well as being at odds with empirical evidence on prices (Giglio et al. 2016), these theories are also unable to speak to the pervasive empirical and experimental evidence on extrapolative beliefs (Smith et al. 1988, Greenwood and Shleifer 2014). Behavioral theories have instead turned to overconfidence and short-sale constraints (Harrison and Kreps 1978, Scheinkman and Xiong

Electronic copy available at: https://ssrn.com/abstract=4666338

2003), and more recently to modeling extrapolative expectations themselves (De Long et al. 1990, Hong and Stein 1999, Glaeser and Nathanson 2017, Bordalo et al. 2021, Liao et al. 2021, Jin and Sui 2022). Following a sequence of positive news, investors extrapolate recent price rises, and become more optimistic. This then translates into even higher prices, and even more optimistic future beliefs. By directly modeling the self-sustaining feedback between outcomes and beliefs that is characteristic of bubbles, these models generate many features of the Kindleberger (1978) narrative.[1]

At the same time, the reduced form nature of extrapolation considered in these theories leaves several questions open. First, what are the foundations of extrapolative expectations, and what determines how strongly traders extrapolate price changes in updating their future beliefs? Second, why is it that "[b]y no means does every upswing in business excess lead inevitably to mania and panic" (Kindleberger 1978)? In other words, why is it that the same type of extrapolative beliefs sometimes leads prices and beliefs to become extreme and decoupled from fundamentals, while in normal times we don't observe such extreme responses to shocks?

To answer these questions we first provide a micro-foundation for the degree of price extrapolation with a theory of "Partial Equilibrium Thinking" (PET) (Bastianello and Fontanier 2023) in which traders fail to realize the general equilibrium consequences of their actions when learning information from prices. Second, consistent with the Kindleberger narrative, we draw a distinction between normal times shocks and displacement shocks, and show that while partial equilibrium thinking leads to constant price extrapolation in normal times, it leads to stronger and time-varying extrapolation following a displacement.

Micro-founding the degree of extrapolation in this way and formalizing the difference between normal times shocks and displacement shocks provides a unifying theory in which

[1] See Brunnermeier and Oehmke (2013), Xiong (2013) and Barberis (2018) for exhaustive surveys on bubbles and crashes, and Hirshleifer (2015) for a broader survey on behavioral finance.

Electronic copy available at: https://ssrn.com/abstract=4666338

the two-way feedback between prices and beliefs is present at all times, but only manifests itself in explosive ways under very specific circumstances. According to Soros (2015): "[...] in most situations [the two-way feedback] is so feeble that it can safely be ignored. We may distinguish between near-equilibrium conditions where certain corrective mechanisms prevent perceptions and reality from drifting too far apart, and far-from equilibrium conditions where a reflexive double-feedback mechanism is at work and there is no tendency for perceptions and reality to come closer together [...]." We formalize this notion of "near-equilibrium" and "far-from equilibrium" conditions by modeling the distinction between normal times shocks which do not generate large changes to the environment, and Kindleberger-type displacements which instead do.

To illustrate our notion of partial equilibrium thinking, consider some investors who see the price of a stock rise, but do not know what caused this. They may think that some other more informed investors in the market received positive news about this stock and decided to buy, pushing up its price. Given this thought process, they infer positive news about it, and also buy, leading to a further price increase. At this point, rational agents perfectly understand that this additional price rise is not due to further good news, but simply reflects the lagged response of uninformed agents who are thinking and behaving just like them. As a result, they no longer update their beliefs in response to this second price rise, and the two-way feedback between prices and beliefs fails to materialize.

However, for uninformed agents to learn the right information from prices, they must perfectly understand what generates the price changes they observe at each point in time, which in turn requires them to perfectly understand all other agents' actions and beliefs. Theories of rational expectations model this level of understanding by assuming common knowledge of rationality, which has been widely rejected by experimental evidence (Crawford et al. 2013). We relax this assumption by building on a large literature in social learning that has documented how people tend to under-estimate the extent to which others also learn from aggregate outcomes (Kübler and Weizsäcker 2004, Penczynski 2017,

Electronic copy available at: https://ssrn.com/abstract=4666338

Eyster et al. 2018, Enke and Zimmermann 2019), and has formalized this behavior with models of correlation neglect, naïve herding, cursedness, and k-level thinking (DeMarzo et al. 2003, Eyster and Rabin 2005, Eyster and Rabin 2010).[2,3,4] We introduce this type of bias in a general equilibrium environment and develop of a dynamic theory of partial equilibrium thinking (Bastianello and Fontanier 2023), whereby "otherwise rational expectations fail to take into account the strength of similar responses by others" (Kindleberger 1978). Specifically, PET agents neglect that all other uninformed agents are thinking and behaving just like them, and they instead attribute any price change they observe to new information alone (DeMarzo et al. 2003). Following the second price rise in our earlier example, PET agents attribute it to further good news, encouraging further buying and inducing further price rises in a self-sustaining feedback between prices and beliefs. In this paper we formalize the intuition behind this example and show how, depending on the information structure, the strength of this feedback effect may be time-varying.

We begin by introducing partial equilibrium thinking into a standard infinite horizon model of a financial market where each period a continuum of investors solve a portfolio choice problem between a risky and a riskless asset. Our agents differ in their ability to observe fundamental news: a fraction of agents are informed and observe fundamental shocks, and the remaining fraction of agents are uninformed and instead infer information from prices. Motivated by empirical and experimental evidence that traders extrapolate trends as opposed to instantaneous price movements (Andreassen and Kraus 1990, Case et al. 2012), we assume that traders learn information from past as opposed to current

[2]See also Bohren (2016), Esponda and Pouzo (2016), Gagnon-Bartsch and Rabin (2016), Fudenberg et al. (2017), Bohren and Hauser (2021), Frick et al. (2020) on misinference in social learning.

[3]Bastianello and Fontanier (2023) considers more general types of misinference, and is informative of how results generalize under different forms of model misspecification.

[4]We contribute to this literature in two ways. First, we introduce this type of bias in a general equilibrium environment, where prices don't only have a purely informational role, but they also have a market feedback effect role, as they act as a measure of scarcity. Second, by drawing a distinction between normal times and displacement shocks, we study how the latter introduce time-variation in the relative strength of the informational and scarcity roles of prices, and show how this allows for reversals even after periods where outcomes and beliefs have become extreme and decoupled from fundamentals..

Electronic copy available at: https://ssrn.com/abstract=4666338

prices, as is standard in models of extrapolative beliefs (De Long et al. 1990, Hong and Stein 2007 and Barberis et al. 2018).[5]

Given this information structure, price changes reflect both the contemporaneous response of informed agents to news, and the lagged response of uninformed agents who learn from past prices. However, when uninformed agents think in partial equilibrium, they neglect the second source of price variation and attribute any price change to new information alone, leading to a simple type of price extrapolation.

We show that the degree of extrapolation and the bias that partial equilibrium thinking generates are decreasing in informed traders' informational edge. This edge is defined as the aggregate confidence of informed traders relative to the aggregate confidence of uninformed traders, and is higher when there are more informed traders in the market, and when the precision of the additional information informed traders hold is higher. When this informational edge is high, informed traders trade more aggressively, and the influence on prices of uninformed traders' beliefs is lower. This leads partial equilibrium thinkers to neglect a smaller source of price variation, therefore leading to a smaller bias and a smaller strength of the feedback between prices and beliefs. Conversely, when informed traders' edge is low, partial equilibrium thinkers neglect a greater source of price variation, leading to a larger bias and a stronger feedback effect. By understanding how this edge varies in response to different types of shocks, we can then understand how following a displacement shock partial equilibrium thinking generates much more amplification than in normal times.

[5] Bastianello and Fontanier (2023) explore the implications of partial equilibrium thinking with learning from current prices and more general types of model misspecification in a static framework. The assumption of learning from past prices is standard in models of extrapolative expectations (De Long et al. 1990, Hong and Stein 2007 and Barberis et al. 2018), and allows us to model the evolution of the two-way feedback between outcomes and beliefs dynamically, as opposed to restricting us to studying the steady state properties of this process. We can therefore think of models of extrapolative beliefs as embedding an additional layer of bounded rationality, which prevents traders from updating their beliefs and trade at the same time, and instead induces them to perform these two tasks sequentially. Notice that for consistency we compare equilibrium outcomes under PET to a rational benchmark which embeds the same assumption of learning from past prices.

Electronic copy available at: https://ssrn.com/abstract=4666338

We show that in normal times informed agents' edge is constant over time. For example, normal times shocks may come in the form of earnings announcements: sophisticated traders are better able to understand the long run implications of such shocks, and uninformed retail traders can learn about them more slowly by observing how the market reacts to such news. When this is the case, informed traders are always one step ahead of uninformed traders, and their edge is high and constant, meaning that partial equilibrium thinkers neglect a small source of price variation, thus leading to weak departures from rationality, as when Soros' notion of "near equilibrium" conditions are satisfied.

This is no longer true following a Kindleberger-type displacement, when the informational edge becomes time-varying. Specifically, displacements are "something new under the sun," and the implications of such shocks for long term outcomes can be learnt only gradually over time.[6] These shocks wipe out much of informed agents' edge as not even the most informed of informed agents are able to immediately grasp the full long-term implications of such events. This leads informed agents to trade less aggressively, and to a rise in the influence on prices of uninformed traders' beliefs. Partial equilibrium thinkers then neglect a greater source of price variation, leading to a stronger bias. This contributes to fuelling the strength of the feedback between prices and beliefs, allowing both to accelerate away from fundamentals, as "far-from equilibrium" conditions take over in determining equilibrium dynamics. As informed traders learn more about the displacement over time, they regain their edge, leading to a gradual fall in the degree of extrapolation, and in the strength of the feedback effect. When the feedback effect runs out of steam, the bubble bursts, and prices and beliefs converge back towards fundamentals. The exact shape of the bubble then depends on the speed with which informed

[6] The distinction that we draw between normal times shocks and displacement shocks is also consistent with recent evidence in Kogan et al. (2023), who show that retail investors behave differently when trading cryptos relative to traditional asset classes. This is consistent with our model, where cryptos are subject to a displacement shock: they are indeed a new technology, and traders may only learn about the likelihood of adoption only gradually over time.

Electronic copy available at: https://ssrn.com/abstract=4666338

traders learn more about the displacement.

Relative to earlier micro-foundations of price extrapolation (Hong and Stein 1999, Malmendier and Nagel 2011, Fuster et al. 2012, Glaeser and Nathanson 2017, Greenwood and Hanson 2015, Jin and Peng 2023), this paper makes three key contributions. First, we draw a distinction between normal times shocks and displacement shocks: consistent with the Kindleberger (1978) narrative, bubbles may only arise following a displacement shock, and we provide a formal way to model this. Second, partial equilibrium thinking provides a micro-foundation for *time-varying* price extrapolation, therefore highlighting an additional source of amplification during the formation of bubbles. Third, we are able to exploit the properties of unstable and non-stationary regions, as displacements make the transition to such regions only temporary. This allows us to offer an explanation for why not *every* large positive shock leads to bubbles and crashes, in a way that is consistent with both historical narratives and more recent empirical evidence (Kindleberger 1978).[7,8]

Finally, Section 3 studies how our bias interacts with inter-temporal trading motives, and show that whether speculators amplify bubbles or arbitrage them away depends on their beliefs of whether mispricing is temporary or not. While the focus of our paper is on the role of higher order beliefs in contributing to misinference, this section allows us to connect to the distinct but complementary literature that has studied the role of higher

[7]This environment-dependent strength of extrapolation distinguishes our work from other learning models, as in Branch and Evans (2011) or Adam et al. (2017), as we discuss in more details in Section 2.3. Additionally, our model features heterogenous beliefs and large trading volume, therefore lending itself to explaining key features of bubbles.

[8]Our model also differs from models of fundamental extrapolation on the same three dimensions mentioned in the text (time-variation in the degree of extrapolation, our formal modeling of displacements and normal times shocks, and how we exploit non-stationary regions). Additionally, unlike models of price extrapolation, models of fundamental extrapolation do not embed a two-way feedback between prices and beliefs, and therefore cannot generate as much amplification (see Bastianello and Fontanier 2023 for further discussion on this point). Specifically, fundamental extrapolation on its own cannot yield non-stationary dynamics, and cannot endogenously generate a convex price path in the run-up of a bubble without relying on a convex path in fundamentals as well (at which point even the rational benchmark leads to convex price dynamics). Our notion of displacements does not rely on convex fundamentals, and instead captures important features of shocks that have historically lead to bubble and crashes (Kindleberger 1978).

Electronic copy available at: https://ssrn.com/abstract=4666338

order beliefs in forecasting (De Long et al. 1990, Abreu and Brunnermeier 2002, Schmidt-Engelbertz and Vasudevan 2023). Misinference regards agents' model of the world in interpreting *past outcomes*. The second type of bias instead regards agents' forward-looking model of the world: how do agents forecast *future equilibrium outcomes* given their information set. These two biases need not bite at the same time, but may interact with each other. The first part of our analysis isolated the role of misinference by having traders forecast fundamentals, therefore shutting down higher order beliefs in forecasting. Conversely, in Section 3 we consider the case where traders forecast future prices, and study how higher order beliefs also affect the behavior of informed traders who are by design not subject to biases in inference. Consistent with previous findings, we show that if informed traders think that they live in a rational world and that mispricing is temporary, they arbitrage the bubble away immediately, and bubbles and crashes do not arise. If instead they realize that other traders are biased, that future mispricing is predictable and that they will be able to sell the asset to "a greater fool" at a higher price in the future, they increase their position in the asset, thus pushing prices up further, and amplifying the bubble. Internet Appendix D shows that this type of amplification is present even when informed traders solve the full inter-temporal problem with dynamic trading motives, as in He and Wang (1995). These predictions are consistent with bubbles being associated with destabilizing speculation (Keynes 1936), and with more sophisticated traders initially riding the bubble (Brunnermeier and Nagel 2004).

The paper proceeds as follows. In Section 1 we introduce our notion of partial equilibrium thinking and study its implications in normal times. Section 2 models displacements and studies how they interact with partial equilibrium thinking in generating bubbles and crashes. Section 3 adds inter-temporal trading motives. Section 4 concludes.

Electronic copy available at: https://ssrn.com/abstract=4666338

1 Normal Times

In this section we introduce our notion of partial equilibrium thinking (PET) in normal times, which we think of as periods where shocks come in the form of regular earning announcements that do not cause significant changes in the composition of traders in the market, or in the relative confidence of traders.

1.1 Setup

We consider an infinite period model, where agents solve a portfolio choice problem between a risk-free and a risky asset.

Assets and fundamentals. The risk-free asset is in elastic supply and we normalize its price and its risk-free rate to one. The risky asset is in fixed net supply Z and pays a liquidating dividend when it dies an uncertain terminal date.[9] In each period, with probability β the asset remains alive and produces $u_t \overset{iid}{\sim} N(0, \sigma_u^2)$ worth of terminal dividends, and with probability $(1-\beta)$ the asset dies, and all accumulated dividends are paid out (Blanchard 1985). Introducing an uncertain terminal date is a simple and effective modeling device that increases tractability by serving two key purposes: it allows us to study partial equilibrium thinking in isolation from horizon effects from approaching a fixed terminal date, and it keeps variances bounded even as we allow the terminal date to be arbitrarily far into the future.[10]

[9]The fixed supply ensures that prices are fully revealing (Grossman 1976). Internet Appendix C allows for the supply of the risky asset to be stochastic, so that prices are only partially revealing (Diamond and Verrecchia 1981). The key intuitions remain unchanged.

[10]Supplementary Material available on the authors' websites considers many alternative processes for the dynamic evolution of the fundamental value of the asset. For example, we consider the case where fundamentals evolve as a random walk with a fixed terminal date, or where the growth rate of fundamentals follows an AR(1). Our results on the interaction of partial equilibrium thinking with different types of shocks are robust to these variations without an uncertain terminal date, and we choose this process for fundamentals for tractability.

Electronic copy available at: https://ssrn.com/abstract=4666338

From the point of view of period t, the asset is still alive in period $t+h$ with probability β^h. Taking expectations over all possible terminal dates, the present value of the terminal dividend in period t, conditional on realized future shocks $\{u_{t+h}\}_{h=1}^{\infty}$, can be written as:[11]

$$\mathbb{E}_t[D_T|\{u_j\}_{j=0}^t, \{u_{t+h}\}_{h=1}^{\infty}] = \bar{D} + \sum_{j=0}^{t} u_j + \sum_{h=1}^{\infty} \beta^h u_{t+h} \tag{1}$$

where $\bar{D} > 0$ is constant and is common knowledge. This expression reflects that from the point of view of period t, the asset has produced $\sum_{j=0}^{t} u_j$ worth of terminal dividends while alive in these first t periods, and with probability β^h the asset is still alive in period $t+h$, and if so it will produce an amount u_{t+h}. This survival probability β then acts as a very natural discount rate such that dividends paid further into the future receive a lower weight today.

Objective function. Our economy is populated by a continuum of measure one of fundamental traders, who have mean-variance utility, and solve the following portfolio choice problem in each period:

$$\max_{X_{i,t}} \quad \left\{ X_{i,t} \left(\mathbb{E}_{i,t}[D_T] - P_t \right) - \frac{1}{2} \mathcal{A} X_{i,t}^2 \mathbb{V}_{i,t}[D_T] \right\} \tag{2}$$

where $X_{i,t}$ is the dollar amount that agent i invests in the risky asset in period t, $\mathcal{A}$ is the coefficient of absolute risk aversion, and $\mathbb{E}_{i,t}[D_T]$ and $\mathbb{V}_{i,t}[D_T]$ refer to agent i's posterior mean and variance beliefs about the fundamental value of the asset conditional on their information set in period t. The corresponding first order condition yields the following standard demand function for the risky asset:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[D_T] - P_t}{\mathcal{A} \mathbb{V}_{i,t}[D_T]} \tag{3}$$

[11]The subscript T stands for Terminal dividend, and not for period T.

Electronic copy available at: https://ssrn.com/abstract=4666338

which is increasing in agent i's expected payoff, and decreasing in the risk they associate with holding the asset.

This objective function allows us to study partial equilibrium thinking in isolation of inter-temporal trading motives. This has two advantages: first, it increases tractability substantially; second, it allows us to study the role of higher order beliefs in inference (which is the focus of our paper) separately from higher order beliefs in forecasting (which has been studied in earlier work).[12] In Section 3 we consider the more common objective function with mean-variance utility over next period wealth, with traders who forecast next period prices as opposed to long-term fundamentals. In Internet Appendix D, we additionally allow informed traders to solve the full intertemporal dynamic trading problem as in He and Wang (1995). The key results and intuitions of how partial equilibrium thinking shapes uninformed traders' beliefs in response to normal times shocks and displacement shocks are unchanged.

Information structure and beliefs. Turning to the information structure, we assume that a fraction ϕ of agents are informed, and observe the fundamental shock u_t in every period. The remaining fraction $(1 - \phi)$ of agents are uninformed and do not observe any of the fundamental shocks, but can learn information from prices.

Given experimental evidence by Andreassen and Kraus (1990) that traders tend to extrapolate recent price trends rather than instantaneous price movements, we assume that traders learn information from past as opposed to current prices, in the spirit of the positive feedback traders in De Long et al. (1990), Hong and Stein (1999), and Barberis et al. (2018).[13]

Importantly, while other details of our setup where chosen for tractability, the asym-

[12] See Abreu and Brunnermeier (2002) among others for a study of the role of higher order beliefs in forecasting. Section 3 of our paper expands on this discussion.

[13] Supplementary Material available on the authors' websites show that the main intuitions of the model go through even if we assume that uninformed traders submit market orders that do not condition on the current price level.

Electronic copy available at: https://ssrn.com/abstract=4666338

metric nature of the information structure, and learning from past as opposed to current prices are key to our model. The first assumption allows informed agents to have an edge relative to uninformed traders, and we think of it as capturing different types of market participants (e.g. hedge funds vs retail traders). The second assumption allows the feedback effect between prices and beliefs embedded in partial equilibrium thinking to play out dynamically rather than in a single period, and is consistent with evidence on extrapolative beliefs.

Equilibrium. To solve the model, we proceed in three steps. First, we solve for the true price function which generates the outcomes that agents observe. Second, we turn to PET agents' beliefs of what generates the prices they observe, which allows us to pin down the mapping that PET agents use to learn information from prices. Finally, we solve the equilibrium recursively, and study the properties of equilibrium outcomes.

1.2 True Price Function in Normal Times

To solve for the true market clearing price function, we need to specify agents' posterior beliefs, compute agents' asset demand functions, and impose market clearing. Starting from agents' beliefs, we know that in period t all informed agents trade on the information they receive, and update their beliefs accordingly:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t-1}[D_T] + u_t \tag{4}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_I \tag{5}$$

Instead, all uninformed agents learn information from past prices. Let $\tilde{u}_{t-1}$ be the fundamental shock which uninformed traders learn from the past price they observe, P_{t-1}. More generally, we denote with a $\tilde{\cdot}$ uninformed traders' beliefs about a variable. In this case, since prices are fully revealing, uninformed traders believe they are extracting

Electronic copy available at: https://ssrn.com/abstract=4666338

from P_{t-1} the exact fundamental shock that informed traders observe in $t-1$, so $\tilde{u}_{t-1}$ is uninformed agents' belief of the $t-1$ fundamental shock, u_{t-1}. For now, we treat $\tilde{u}_{t-1}$ as a generic signal uninformed traders learn from past prices, and we derive this as an equilibrium object in the next section where we explicitly solve for the inference problem.[14] We can write uninformed traders' posterior beliefs as:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} \tag{6}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{U,t}\left[u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h}\right] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \mathbb{V}_U \tag{7}$$

Importantly, comparing (5) and (7) shows that informed traders have an edge relative to uninformed traders. While informed traders always face uncertainty over all future fundamental shocks, uninformed traders additionally face uncertainty over the current shock, as they only learn information from past prices. Specifically, we define the aggregate informational edge of informed traders (ζ) as the aggregate confidence of informed traders relative to the aggregate confidence of uninformed traders:

$$\zeta \equiv \frac{\phi}{(1-\phi)} \frac{\tau_I}{\tau_U} \tag{8}$$

where $\tau_i \equiv (\mathbb{V}_i)^{-1}$ is the confidence of agent $i \in \{I, U\}$. This edge is increasing in the fraction of informed traders (ϕ), and in the relative individual level confidence of informed and uninformed traders (τ_I/τ_U). Since in normal times ϕ and τ_I/τ_U are constant, the informational edge is also constant.

Given agents' posterior beliefs, we can compute their asset demand functions and impose market clearing to find that prices are a weighted average of agents' beliefs minus

[14] Whether $\tilde{u}_{t-1} = u_{t-1}$ or $\tilde{u}_{t-1} \neq u_{t-1}$ depends on the mapping uninformed traders use to extract information from prices. In Sections 1.3 and 1.4 we show that if traders have rational expectations, then $\tilde{u}_{t-1} = u_{t-1}$, but if instead they use a misspecified mapping, as with partial equilibrium thinking, they extract biased information from prices and $\tilde{u}_{t-1} \neq u_{t-1}$.

Electronic copy available at: https://ssrn.com/abstract=4666338

a risk premium component that compensates them for bearing risk:

$$P_t = a\mathbb{E}_{I,t}[D_T] + b\mathbb{E}_{U,t}[D_T] - c \tag{9}$$

where:

$$a \equiv \frac{\zeta}{1+\zeta} \qquad b \equiv \frac{1}{1+\zeta} \tag{10}$$

and $c \equiv \frac{\mathcal{A}Z}{\phi\tau_I + (1-\phi)\tau_U}$. The expressions in (10) then show that the influence on prices of informed (uninformed) agents' beliefs is increasing (decreasing) in informed agents' informational edge. Taking first differences of the price function in (9) and of agents' beliefs in (4) and (8) we find that price changes reflect both informed traders' instantaneous response to shocks, and uninformed agents' lagged response:

$$\Delta P_t = \underbrace{au_t}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{b\tilde{u}_{t-1}}_{\substack{\text{lagged response of } U \\ \text{from learning from past prices}}} \tag{11}$$

To specify what information uninformed agents extract from past prices we need to understand what uninformed agents think is generating the price changes that they observe. In what follows we first explore the inference problem under rational expectations, and we then turn to partial equilibrium thinking.

1.3 Rational Expectations Benchmark

If uninformed traders have rational expectations, they perfectly understand that (11) generates the price changes they observe, and are therefore able to infer the right information

Electronic copy available at: https://ssrn.com/abstract=4666338

from prices:[15]

$$\tilde{u}_{t-1} = u_{t-1} \tag{12}$$

However, for uninformed agents understand the mapping in (11), they must perfectly understand other agents' actions and beliefs. In what follows, we relax this assumption.

1.4 Partial Equilibrium Thinking

When agents think in partial equilibrium, they misunderstand what generates the price changes that they observe because they fail to realize the general equilibrium consequences of their actions (Bastianello and Fontanier 2023). The way that PET manifests itself in this setup is that all agents learn information from prices, but they fail to realize that other agents do too.

Formally, PET agents think that in period $t-1$ informed agents update their beliefs with the new fundamental information received by informed agents, $\tilde{u}_{t-1}$:[16]

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} \tag{13}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_I \tag{14}$$

On the other hand, they think that all other uninformed agents do not learn information from prices, and instead trade on the same unconditional prior beliefs they held in

[15] To keep this rational benchmark as close as possible to our notion of partial equilibrium thinking, we restrict uninformed rational traders to also learn information from *past* prices. Internet Appendix B.1 explicitly solves for this rational benchmark.

[16] The use of $t-1$ subscripts instead of t is to highlight that uninformed agents learn information from past prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

Electronic copy available at: https://ssrn.com/abstract=4666338

period $t = 0$:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D] = \bar{D} \tag{15}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_U \tag{16}$$

where the equivalences in (14) and (16) highlight that in normal times, PET agents understand that all agents face constant uncertainty over time.[17]

Given these beliefs, PET agents think that the equilibrium price in period $t-1$ is given by:

$$P_{t-1} = \tilde{a}\tilde{\mathbb{E}}_{I,t-1}[D_T] + \tilde{b}\tilde{\mathbb{E}}_{U,t-1}[D_T] - \tilde{c} \tag{17}$$

where:

$$\tilde{a} \equiv \frac{\tilde{\zeta}}{1+\tilde{\zeta}} \qquad \tilde{b} \equiv \frac{1}{1+\tilde{\zeta}} \tag{18}$$

and where $\tilde{c} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_I + (1-\phi)\tilde{\tau}_U}$ and $\tilde{\zeta} \equiv \frac{\phi}{1-\phi}\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$.[18] Taking first differences of the perceived price function in (17), and of uninformed agents' perceptions of other agents' beliefs in (13) and (15):

$$\Delta P_{t-1} = \underbrace{\tilde{a}\tilde{u}_{t-1}}_{\substack{\text{instantaneous response of } I\\ \text{to new information}}} \tag{19}$$

which shows that when agents think in partial equilibrium they attribute any price change they observe to new information alone. In so doing, they neglect the second source of price variation in (11), which is due to the lagged response of all other uninformed traders.

[17]Moreover, since $\tilde{\mathbb{V}}_I = \mathbb{V}_I < \tilde{\mathbb{V}}_U = V_U$, PET agents are not misspecified about other agents' second moment beliefs, and they understand that informed agents have an informational edge.

[18]From (14) and (16), we see that $\tilde{\tau}_I = \tau_I$ and $\tilde{\tau}_U = \tau_U$, so that in normal times $\tilde{\zeta} = \zeta$. However, displacement shocks draw a wedge between $\tilde{\zeta}$ and ζ, so we distinguish between these two quantities from the outset.

Electronic copy available at: https://ssrn.com/abstract=4666338

PET agents then invert the mapping in (19) to extract $\tilde{u}_{t-1}$ from prices:[19]

$$\tilde{u}_{t-1} = \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{21}$$

Therefore, PET provides a micro-foundation for extrapolative expectations as uninformed traders extract a positive signal and become more optimistic whenever they see a price rise, and extract a negative signal and become more pessimistic whenever they see a price fall. This is unlike the rational expectations benchmark, where uninformed traders become more optimistic (pessimistic) following a price rise (fall) *only* if that price change is due to new information. If the price change they observe is instead due to the lagged response of uninformed traders who are learning information from past prices, rational traders do not update their beliefs.

The bias inherent in partial equilibrium thinking is then increasing in the source of price variation they neglect, which, in turn, is decreasing in informed traders' informational edge. Intuitively, a lower edge (from a smaller fraction of informed traders in the market, or from a lower confidence of informed relative to uninformed traders) increases the influence on prices of uninformed agents' beliefs, leading PET agents to omit a greater source of price variation.

[19]We can compare this to the mapping used by rational uninformed traders, who understand that (11) generates the price function they observe, and therefore use the following mapping to infer information from prices (as further discussed in Internet Appendix B.1):

$$\tilde{u}_{t-1} = \left(\frac{1}{a}\right) \Delta P_{t-1} - \left(\frac{b}{a}\right) \tilde{u}_{t-2} \tag{20}$$

Since in normal times $\tilde{a} = a$, comparing (20) and (21) makes clear that the bias inherent in partial equilibrium thinking doesn't come directly from the weight that uninformed traders put on past price changes (which is $1/a$ in both the rational and PET case), but rather from neglecting the part of the price variation that comes from the lagged response of all other uninformed traders. In particular, notice that it is rational to put less weight on price changes when informed traders' edge is higher: when this is the case, information is incorporated more strongly into prices, so that traders have to extrapolate less strongly to recover that information. Internet Appendix C extends this discussion to the case where prices are only partially revealing.

Electronic copy available at: https://ssrn.com/abstract=4666338

Proposition 1 (Micro-foundation of Price Extrapolation)**.** *Partial equilibrium thinking provides a micro-foundation for extrapolative expectations:*

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{22}$$

where $\frac{1}{\tilde{a}} = 1 + \frac{1}{\tilde{\zeta}}$*. Moreover, given a one-off shock to fundamentals, the bias is decreasing in the true and perceived informational edge of informed traders:*

$$\tilde{u}_{t-1} - u_{t-1} = \left(\frac{b}{\tilde{a}}\right) \tilde{u}_{t-2} \tag{23}$$

where $\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1 + \frac{1}{\tilde{\zeta}}\right)$.

Proof. All proofs are in Appendix A. □

1.5 The Feedback-Loop Theory of Bubbles

Combining the expressions of the true price function in (11) and of the extracted signal in (21), we find that when traders think in partial equilibrium changes in prices and in beliefs evolve as an AR(1):

$$\tilde{u}_{t-1} = u_{t-1} + \left(\frac{b}{\tilde{a}}\right) \tilde{u}_{t-2} \tag{24}$$

$$\Delta P_t = a u_t + \left(\frac{b}{\tilde{a}}\right) \Delta P_{t-1} \tag{25}$$

This is in contrast to the rational benchmark where, combining (11) and (20), we find that when traders are rational price changes evolve as an MA(1):

$$\tilde{u}_{t-1} = u_{t-1} \tag{26}$$

$$\Delta P_t = a u_t + b u_{t-1} \tag{27}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

Intuitively, partial equilibrium thinkers mistakenly infer a sequence of shocks from a one-off shock, and this leads to over-reaction, as is clear from the presence of the second term in (24) which is instead absent in the rational counterpart in (26). Following a one-off shock, PET agents fail to realize that the second price rise is due to the buying pressure of all other uninformed agents, and instead attribute it to further good news, which in turn fuels even higher prices and more optimistic beliefs, in a self-sustaining feedback loop, just as we saw in the example in the introduction.

1.5.1 Strength of the Feedback Effect

The AR(1) coefficient in the processes that describe changes in equilibrium prices and beliefs in (24) and (25) is key to determining the properties of equilibrium outcomes. In our case, this coefficient also has a special meaning, in that it captures the strength of the feedback between prices and beliefs, and it is decreasing both in the true informational edge (ζ), and in uninformed agents' perception of it ($\tilde{\zeta}$):

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) \tag{28}$$

Intuitively, when uninformed agents' perception of the informational edge is low, they neglect a greater source of price variation, leading to a greater bias. Moreover, when the true informational edge of informed agents is low, the influence on prices of uninformed traders' biased beliefs is higher. Both these forces contribute to fuelling the feedback between outcomes and beliefs.

Proposition 2 (Strength of the Feedback Effect)**.** *When agents think in partial equilibrium, the strength of the feedback between outcomes and beliefs is decreasing both in the true informational edge* (ζ)*, and in uninformed agents' perception of it* ($\tilde{\zeta}$)*. Specifically, environments with a smaller fraction of informed traders* (ϕ)*, and with a lower true and perceived confidence of informed agents relative to uninformed agents* (τ_I/τ_U, $\tilde{\tau}_I/\tilde{\tau}_U$) *are*

Electronic copy available at: https://ssrn.com/abstract=4666338

characterized by a stronger feedback between prices and beliefs.

Empirical Predictions. Equation (24) shows that deviations from rationality are increasing in the strength of the feedback effect, leading to the following empirical prediction both in the cross-section, and over time.

Proposition 3 (Deviations from Rationality)**.** *Deviations from rationality in both prices and beliefs are decreasing in the true and perceived informational edges* (ζ, $\tilde{\zeta}$)*. Specifically, following a one-off shock to fundamentals, environments with a smaller fraction of informed agents* (ϕ)*, and with a lower true and perceived confidence of informed agents relative to uninformed agents* ($\tau_I/\tau_U, \tilde{\tau}_I/\tilde{\tau}_U$) *exhibit greater departures from rationality.*

Prior work has used a number of proxies for the fraction of informed agents and for the informativeness of news (see Veldkamp (2023) for a review), and these proxies can be used to test our predictions empirically. For example, Gompers and Metrick (2001) and Yan and Zhang (2009) use the share of institutional investors to proxy for informed traders, while Laarits and Sammon (2022) use the fraction of retail traders as a proxy for uninformed trading.[20] Turning to the precision of new information, Hong et al. (2000) proxy this with the number of analysts covering a given stock, while Bae et al. (2008) uses the precision of forecasts reported in survey data.

Consistently with our theory that uninformed traders extrapolate more strongly when the informational edge is lower, Hong et al. (2000) find that, holding size fixed, momentum strategies work better among stocks with lower analyst coverage. Similarly, Andrade et al. (2013) study the 2007 bubble episode in China and find significantly smaller bubbles in stocks for which there is greater analyst coverage. More recently, Kogan et al. (2023) find that retail traders engage in very different trading behavior in cryptos (where the share of informed traders is arguably lower) relative to stocks and gold.

[20] Da et al. (2021) find that a proxy of retail investors' expectations negatively predicts future returns, and more so among stocks with low institutional ownership and a high degree of extrapolation.

Electronic copy available at: https://ssrn.com/abstract=4666338

Finally, in Proposition 4 we show how our theory predicts stronger and time-varying extrapolation following a displacement shock when informed traders lose their aggregate edge, consistent with suggestive evidence in Cassella and Gulen (2018) and Bybee (2023) who find stronger extrapolation during the formation of bubbles.

1.5.2 Stable and Unstable Regions

Another feature of the AR(1) processes in (24) and (25) is that the system can be stationary or non-stationary, depending on whether $b/\tilde{a} < 1$ or $b/\tilde{a} > 1$. When $b/\tilde{a} < 1$, changes in prices and in beliefs in (24) and (25) are stationary, and shocks eventually die out, so that prices and beliefs exhibit momentum and converge to a new steady state. On the other hand, when $b/\tilde{a} > 1$ the system is non-stationary and the influence of the feedback effect is explosive: consecutive changes in prices and beliefs get larger and larger, and prices and beliefs accelerate in a convex way, becoming extreme and decoupled from fundamentals.

As long as the feedback effect between prices and beliefs is *constant*, the response of prices and beliefs to shocks is either always stationary and convergent, or it is always non-stationary and explosive. Since we do not observe unbounded prices and beliefs in response to normal times shocks (e.g. following earnings announcements), it is plausible to assume that in normal times changes in prices and beliefs are stationary. For this to be the case, it must be that in normal times the aggregate confidence of informed agents is greater than the aggregate confidence of uninformed agents:[21]

$$\frac{b}{\tilde{a}} = \frac{1}{\zeta} < 1 \iff \zeta > 1 \iff \phi\tau_I > (1-\phi)\tau_U \tag{29}$$

In the next section, we show how displacements can generate *time-variation* in the strength of the feedback effect, and shift the economy across stable and unstable regions.

[21] The first equality follows from the fact that in (14) and (16) we saw that $\tau_i = \tilde{\tau}_i$ for $i \in \{I, U\}$, so that $\tilde{\zeta} = \zeta$. Substituting this in (28) yields (29).

Electronic copy available at: https://ssrn.com/abstract=4666338

By bringing the explosive properties of unstable regions into play before the convergent properties of stable regions take over again, displacements can lead to the formation of accelerating bubbles and endogenous crashes (Greenwood et al. 2019).

2 Displacements

A "[d]isplacement is some outside event that changes horizons, expectations, profit opportunities, behavior – some sudden advice many times unexpected. Each day's events produce some changes in outlook, but few significant enough to qualify as displacements" (Kindleberger 1978). Examples include the widespread adoption of a ground-breaking discovery, such as railroads in the 1840s, the radio and automobiles in the 1920s, and the internet in the 1990s; financial liberalization in Japan in the 1980s; or financial innovations such as securitization prior to the 2008 financial crisis (Aliber and Kindleberger 2015).

While the exact nature of the displacement varies from one bubble episode to another, what these shocks have in common is that they represent "something new under the sun," and their full implications for long term outcomes can only be understood gradually over time, as more information becomes available (Pástor and Veronesi 2009). When the internet was first made available to the public in 1993, investors were aware of this new technology, but at the time nobody knew the full potential of this invention. The development of blockchains as decentralized ledgers has paved the way for cryptocurrencies. However, we are yet to learn about the full implications of this technology or the likelihood of their future adoption, and cryptos have indeed been prone to bubbly behavior, as recently documented in Kogan et al. (2023). Moreover, historical narratives also associate displacements with periods characterized with large changes in the compositions of traders in the market, with retail investors playing a prominent role (Aliber and Kindleberger 2015).

Electronic copy available at: https://ssrn.com/abstract=4666338

This is in stark contrast to normal times shocks, which may come in the form of regular earnings announcements. These are not generally associated with either large swings in the composition of traders in the market, nor with stark changes in investors' relative confidence levels. Following these news events, sophisticated traders are well trained to immediately process and understand the content of such news (e.g. the implications of same store sales on long term outcomes), while uninformed traders can learn about their implications more slowly, by seeing how the market reacts to them. As we saw in Section 1, in normal times informed traders are always one step ahead of uninformed traders, and their informational edge is constant.

From a modeling point of view, we can capture displacements as shocks that generate time-variation in either the composition of traders in the market, or in the relative confidence of informed and uninformed traders. We model displacements as being "something new under the sun", which can alter the relative confidence of informed and uninformed traders, and we discuss alternative ways of modeling displacement shocks in Section 2.6.

In this section we show how displacement shocks generate time-variation in informed agents' edge, which in turn leads to time-varying extrapolation, and a time-varying strength of the feedback between prices and beliefs. This can shift the economy between stable and unstable regions. Specifically, when the displacement first materializes, informed agents' edge is wiped out, thus increasing the influence on prices of uninformed agents' beliefs and the strength with which they extrapolate. Both of these forces fuel the feedback between prices and beliefs. If the uncertainty associated with the displacement is high and persistent enough, the economy can enter the unstable region, leading prices and beliefs to accelerate away from fundamentals. Then, as informed agents learn about the new technology and regain their edge, the feedback effect weakens, and the economy re-enters the stable region. This leads the bubble to burst and prices and beliefs to return back towards fundamentals.

We conclude this section by discussing how the speed of information arrival shapes

Electronic copy available at: https://ssrn.com/abstract=4666338

the duration and amplitude of bubbles, and other ways of modeling a displacement.

2.1 Displacement Shocks

We model displacements as an uncertain positive shock to long-term outcomes that agents can learn about only gradually over time. Starting from a normal-times steady state where uninformed agents' beliefs are consistent with the price they observe, in period $t = 0$ both informed and uninformed traders learn that there is "something new under the sun," but do not know the exact implications of such shock for long-term outcomes. Specifically, in period $t = 0$, all agents learn that the terminal dividend changes by an uncertain amount $\omega \sim N(\mu_0, \tau_0^{-1})$, where $\mu_0 > 0$:[22]

$$D_T = \bar{D} + \sum_{j=0}^{\infty} \beta^j u_j + \omega \tag{30}$$

Initially, all agents share the same unconditional prior over ω. Starting in period $t = 1$, each period informed agents observe a common signal that is informative about the displacement, $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim^{iid} N(0, \tau_s^{-1})$. Uninformed agents do not observe these signals but still learn information from past prices.

We solve the model using the same three steps we used in normal times: first, we specify what truly generates price changes agents observe. Second, we specify what uninformed agents think is generating these price changes, and find the mapping PET agents use to extract information from prices. Third, we solve the model recursively, and discuss the properties of equilibrium outcomes.

[22] In Section 2.5 we also consider the case where $\mu_0 < 0$, and show how with partial equilibrium thinking negative bubbles are dampened relative to positive ones.

Electronic copy available at: https://ssrn.com/abstract=4666338

2.2 True Price Function following a Displacement

Following a displacement, informed agents observe new signals u_t and s_t in each period, and they revise their beliefs via standard Bayesian updating:

$$\mathbb{E}_{I,t}[D_T] = \mathbb{E}_{I,t-1}[D_T] + u_t + w_t \tag{31}$$

$$\mathbb{V}_{I,t}[D_T] = \mathbb{V}_{I,t}\left[\sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{V}_I + (t\tau_s + \tau_0)^{-1} \tag{32}$$

where $w_t \equiv \mathbb{E}_{I,t}[\omega] - \mathbb{E}_{I,t-1}[\omega] = \frac{\tau_s}{t\tau_s+\tau_0}\left(s_t - \mathbb{E}_{I,t-1}[\omega]\right)$ is informed agents' revision of their beliefs about the displacement ω in light of the new signal s_t.

On the other hand, in each period t, uninformed agents learn $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ from the price change they observe in period $t-1$, and their posterior beliefs are:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1} \tag{33}$$

$$\mathbb{V}_{U,t}[D_T] = \mathbb{V}_{U,t}\left[u_t + \sum_{h=1}^{\infty} \beta^h u_{t+h} + \omega\right] = \mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1} \tag{34}$$

Importantly, (32) and (34) show that following a displacement informed traders' edge becomes time-varying:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}}\right) \tag{35}$$

Initially, informed agents lose their edge (all agents are just as clueless about the displacement), and they then gradually regain it, as also shown in Figure 1a.

Given these beliefs, we find that, following a displacement, price changes capture both

Electronic copy available at: https://ssrn.com/abstract=4666338

changes in mean beliefs and changes in confidence levels:[23]

$$\Delta P_t = \underbrace{a_t\left(u_t + w_t\right)}_{\substack{\text{instantaneous response of } I\\ \text{to new information}}} + \underbrace{b_t\left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right)}_{\substack{\text{lagged response of } U\\ \text{from learning from past prices}}} + \underbrace{\left(P_{t|t-1} - P_{t-1}\right)}_{\text{changes in confidence}} \tag{37}$$

where:

$$\left(P_{t|t-1} - P_{t-1}\right) \equiv \underbrace{\Delta a_t \mathbb{E}_{I,t-1}[D_T] + \Delta b_t \mathbb{E}_{U,t-1}[D_T]}_{\substack{\text{change in relative weight on}\\ \text{I and U traders' beliefs}}} - \underbrace{\Delta c_t}_{\substack{\text{changes in}\\ \text{risk premium}}} \tag{38}$$

and where $a_t \equiv \frac{\zeta_t}{1+\zeta_t} = 1 - b_t$, $b_t \equiv \frac{1}{1+\zeta_t}$ and $c_t \equiv \frac{\mathcal{A}Z}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}$ are defined as in normal times, but are now time-varying.

Equation (37) shows that price changes now reflect three components. The first two components are due to changes in mean beliefs of both informed and uninformed traders, just as in normal times. However, displacements now bring into play a third source of price variation, which is due to changes in informed and uninformed traders' relative confidence levels. As shown in the definition of $(P_{t|t-1} - P_{t-1})$ in (38), changes in relative confidence levels manifest themselves in two ways. First, changes in relative confidence levels lead to a change in the relative weights on informed and uninformed traders' beliefs (Δa_t and Δb_t), thus leading to changes in the average belief, even holding individual level beliefs fixed. Second, changes in confidence levels also lead to changes in the aggregate risk-bearing capacity, therefore adding an additional source of price variation via changes in the risk premium component (Δc_t).

[23]Market clearing yields:

$$P_t = a_t \mathbb{E}_{i,t}[D_T] + b_t \mathbb{E}_{U,t}[D_T] - c_t \tag{36}$$

where a_t, b_t, and c_t are defined in the main text. Taking first differences of this expression, using agents' posterior beliefs in (31) and (33), and rearranging yields the expression in (37).

Electronic copy available at: https://ssrn.com/abstract=4666338

2.3 Micro-founding Time-varying Price Extrapolation

Just as we did in Section 1, to understand what information uninformed agents extract from past prices, we start by specifying what uninformed agents think is generating the price changes they observe. This, in turn, requires us to work out PET agents' beliefs about other agents' actions and beliefs. Following a displacement, PET agents think that in period $t-1$ informed agents trade on all signals they have received up until period $t-1$, $\{\tilde{u}_j\}_{j=0}^{t-1}$ and $\{\tilde{s}_j\}_{j=1}^{t-1}$:

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} + \tilde{w}_{t-1} \tag{39}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1} \tag{40}$$

where $\tilde{w}_t \equiv \tilde{\mathbb{E}}_{I,t}[\omega] - \tilde{\mathbb{E}}_{I,t-1}[\omega] = \frac{\tau_s}{t\tau_s+\tau_0}\left(\tilde{s}_t - \tilde{\mathbb{E}}_{I,t-1}[\omega]\right)$.

Moreover, PET agents think that all other uninformed agents do not learn information from prices, and instead trade on their fixed prior beliefs:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D_T] = \bar{D} + \mu_0 \tag{41}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \mathbb{V}_U + (\tau_0)^{-1} \tag{42}$$

where (42) shows that following a displacement PET agents believe that other uninformed agents face greater and constant uncertainty as they do not learn new information after the displacement is announced. Combining (40) and (42), we see that PET agents' perception of informed agents' edge ($\tilde{\zeta}_{t-1}$) is initially diluted by the displacement's increase in aggregate uncertainty, and it then gradually rises over time as informed agents learn more about it:

$$\tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}\right) \tag{43}$$

Given these beliefs, PET agents think that following a displacement price changes

Electronic copy available at: https://ssrn.com/abstract=4666338

only reflect two components (rather than three components as in (37)), as they once again neglect that uninformed traders are also learning information from prices:[24]

$$\Delta P_{t-1} = \underbrace{\tilde{a}_{t-1}\left(\tilde{u}_{t-1} + \tilde{w}_{t-1}\right)}_{\substack{\text{instantaneous response of } I \\ \text{to new information}}} + \underbrace{\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)}_{\text{changes in confidence}} \tag{45}$$

where $\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)$ captures changes in prices due to changes in confidence levels:

$$\left(\tilde{P}_{t-1|t-2} - P_{t-2}\right) \equiv \underbrace{\left(\Delta\tilde{a}_{t-1}\tilde{E}_{I,t-2}[D_T] + \Delta\tilde{b}_{t-1}\tilde{\mathbb{E}}_{U,t-2}[D_T]\right)}_{\substack{\text{change in relative weight on} \\ \text{I and U traders' beliefs}}} - \underbrace{\Delta\tilde{c}_{t-1}}_{\substack{\text{changes in} \\ \text{risk premium}}} \tag{46}$$

PET agents then invert the mapping in (45), and attribute the unexpected part of the price change they observe to new information $(\tilde{u}_{t-1} + \tilde{w}_{t-1})$, leading to *time-varying extrapolation.*

Proposition 4 (Time-varying Extrapolation)**.** *Following a displacement shock, partial equilibrium thinking leads to time-varying price extrapolation, with traders extrapolating the unexpected part of the price change they observe. Posterior beliefs are given by:*

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \frac{1}{\tilde{a}_{t-1}}\left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{47}$$

where $\frac{1}{\tilde{a}_{t-1}} = 1 + \frac{1}{\tilde{\zeta}_{t-1}}$.

As well as being consistent with empirical evidence that documents a time-varying extrapolation parameter (Cassella and Gulen 2018, Bybee 2023), micro-founding the ex-

[24]The perceived market clearing condition yields:

$$P_t = \tilde{a}_t\tilde{\mathbb{E}}_{i,t}[D_T] + \tilde{b}_t\tilde{\mathbb{E}}_{U,t}[D_T] - \tilde{c}_t \tag{44}$$

where $\tilde{a}_t$, $\tilde{b}_t$, and $\tilde{c}_t$ are defined in the main text. Taking first differences of this expression, using agents' posterior beliefs in (39) and (41), and rearranging yields the expression in (37). Notice in particular that uninformed traders think other uninformed traders never update their beliefs, so this term does not show up in (45).

Electronic copy available at: https://ssrn.com/abstract=4666338

trapolation parameter in this way allows us to understand the assumptions implicit in models of constant price extrapolation. Specifically, they assume that following a large structural break in prices, agents still forecast prices in exactly the same way as they did before the structural break, which is counterfactual.

This also highlights another important point. We model partial equilibrium thinking by staying as close as possible to the rational expectations benchmark. While the inference problem is much simpler than the rational counterpart (since PET agents do not have to think about higher-order beliefs) it still requires some degree of sophistication on the part of uninformed traders. On the one hand, this is inherent in the nature of our bias, where traders think they are the only ones learning information from prices, and think they have an edge relative to their peers.[25] On the other hand, the reduced form nature of our bias translates into a very simple strategy and heuristic, which does not require much sophistication. If traders think about what generates the price changes they are learning from, it is natural for them to engage in constant price extrapolation when the properties of the environment they are learning from are stable, and to adjust the degree of extrapolation in response to a structural break. In other words, our theory can be understood as explaining when and why agents change heuristics: they do so in response to different types of shocks that change the properties of the environment.[26]

[25] Partial equilibrium thinking can either be seen as an example of the Lake-Wobegan (or better-than-average) effect (Svenson 1981), or as agents paying limited attention to others' informational inferences, rather than having false beliefs about others' inference (Eyster and Rabin 2010).

[26] This is the main distinguishing feature of our model relative to learning models where agents forecast prices using some law of motion (Marcet and Sargent 1989, Evans and Honkapohja 1999, Adam and Marcet 2011). For instance, in Adam et al. (2017), agents know the fundamental process but forecast future prices according to constant-gain learning:

$$\mathbb{E}_t\left[\frac{P_{t+1}}{P_t}\right] = (1-g)\mathbb{E}_{t-1}\left[\frac{P_t}{P_{t-1}}\right] + g\left(\frac{P_{t-1}}{P_{t-2}}\right) \tag{48}$$

This is similar in spirit to the expression we derived in Equation (47). The key difference is that we microfound the degree of extrapolation, which in our model depends on the *properties* of the environment. This allows us to explain why not all shocks to price growth lead to extreme responses, and which ones do.

Electronic copy available at: https://ssrn.com/abstract=4666338

2.4 Displacement, Bubbles and Crashes

By combining the results from Sections 2.2 and 2.3, we find that following a displacement PET agents' prices and beliefs evolve as follows:

$$\Delta P_t = a_t(u_t + w_t) + \left(\frac{b_t}{\tilde{a}_{t-1}}\right)\Delta P_{t-1} - \left(\left(\frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\tilde{P}_{t-1|t-2} - P_{t-2}) - (P_{t|t-1} - P_{t-1})\right) \quad (49)$$

$$(\tilde{u}_{t-1} + \tilde{w}_{t-1}) = \left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(u_{t-1} + w_{t-1}) + \left(\frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\tilde{u}_{t-2} + \tilde{w}_{t-2}) - \frac{1}{\tilde{a}_{t-1}}\left(\tilde{P}_{t-1|t-2} - P_{t-1|t-2}\right) \quad (50)$$

These expressions are reminiscent of the AR(1) processes in (24) and (25), with two key differences, which together allow for the formation of bubbles and crashes following a displacement shock, as shown in Figure 2. First, the strength of the feedback between prices and beliefs is now time-varying, so that equilibrium dynamics can now shift across stable and unstable regions. When the equilibrium dynamics shift to a non-stationary region, prices and beliefs accelerate away from fundamentals leading to the build up of the bubble.[27] Second, the last term in both (49) and (50) acts as a pull-back force, that dampens increases in prices and beliefs during the formation of the bubble. It is this term that ultimately allows uninformed agents' beliefs to be disappointed at the peak of the bubble, leading to reversals and a crash. We now discuss both of these differences in detail.

Starting from the strength of the feedback effect, it now takes the following form:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1 + \frac{1}{\tilde{\zeta}_t}\right) \quad (51)$$

[27] This is why the PET price path in Figure 2 is convex at first. The distinguishing feature of our micro-foundation is that it allows for non-stationary dynamics without relying on a convex path of fundamentals.

Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 1b shows that following a displacement the strength of the feedback effect initially increases as both the true and the perceived informational edges are diluted, and then gradually declines as informed traders eventually regain their edge.[28] Starting from a stable region, if the increase in uncertainty generated by the displacement is large enough, the economy enters an unstable region ($b_t/\tilde{a}_t > 1$), before returning to a stable one ($\lim_{t\to\infty} b_t/\tilde{a}_t < b/\tilde{a} < 1$).[29]

Proposition 5 (Time-varying Strength of the Feedback Effect)**.** *When agents think in partial equilibrium, the strength of the feedback effect between prices and beliefs becomes time varying in response to a displacement shock. In each period t, it is decreasing both in the true informational edge (ζ_t), and in uninformed agents' perception of it ($\tilde{\zeta}_t$). In the long-run, the feedback effect converges to a steady-state value strictly lower than 1.*

While non-stationary regions allow prices and beliefs to become extreme and decoupled from fundamentals, a time-varying strength of the feedback effect is not enough to lead to the bursting of the bubble. Indeed, we need uninformed agents to infer *negative* information from prices ($\tilde{u}_{t-1}+\tilde{w}_{t-1} < 0$) and price changes to become negative ($\Delta P_t < 0$) for prices and beliefs to revert back towards fundamentals and for the bubble to burst. Moving from an unstable to a stable region simply ensures that price *changes* go from being positive and increasing over time to positive and decreasing over time, but does not deliver *negative* price changes on its own.[30] Instead, to achieve the reversal, we need

[28]Using beliefs generated through Large Language Models, Bybee (2023) offers evidence that extrapolation is time-varying and heightened during price run-ups that then lead to crashes.

[29]In the long run the economy always returns to a stable region, as $\lim_{t\to\infty} b_t/\tilde{a}_t < b/\tilde{a} < 1$ since $\lim_{t\to\infty}(b_t - b) = 0$ and $\lim_{t\to\infty}(\tilde{a}_t - \tilde{a}) > 0$, where the last inequality follows from the fact that $\lim_{t\to\infty} \tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U+(\tau_0)^{-1}}{\mathbb{V}_I}\right) > \tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_I}\right)$.

[30]In other words, a time-varying $b_t/\tilde{a}_{t-1}$ would not be enough to get a reversal if equilibrium price changes evolved as follows:

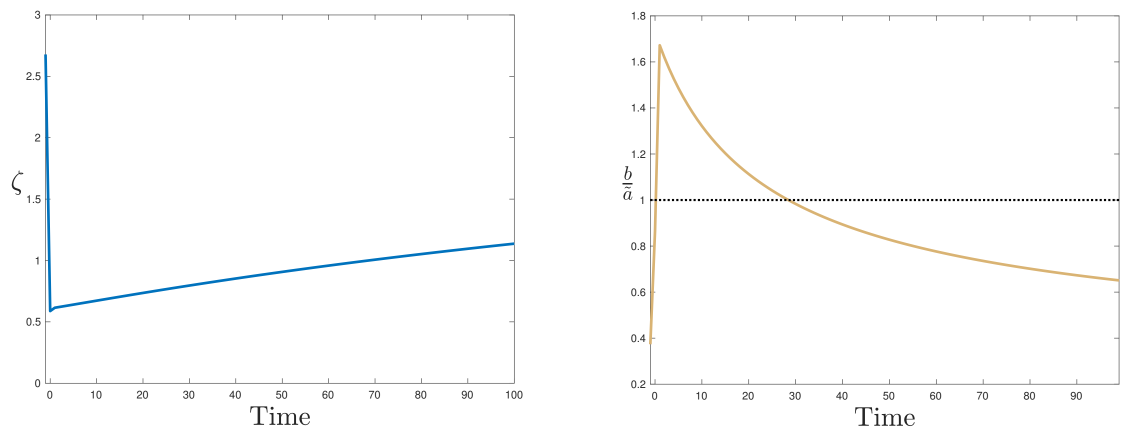
$$\Delta P_t = a_t(u_t + w_t) + \left(\frac{b_t}{\tilde{a}_{t-1}}\right)\Delta P_{t-1} \tag{52}$$

Following a one-off positive shock to fundamentals ($u_t + w_t > 0$ for $t = 0$ and $u_t + w_t = 0$ for $t > 0$), there would be no term that allows for ΔP_t to become negative, unlike the additional term in (49).

Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 1: Time variation in informed traders' edge and in the strength of the feedback effect following a displacement. The dotted line at $^b/_{\tilde{a}} = 1$ on the right panel separates the stable region ($^b/_{\tilde{a}} < 1$) from the unstable region ($^b/_{\tilde{a}} > 1$). Starting from a normal times steady state, a displacement is announced in period $t = 0$. This leads informed traders to lose their edge and the strength of the feedback effect to initially rise. Then, as informed traders gradually regain their edge, the strength of the feedback decline over time. The initial increase in $^b/_{\tilde{a}}$ is increasing in the uncertainty associated with the displacement $(\tau_0)^{-1}$.

(a) Informational edge, ζ (b) Strength of the Feedback Effect, $\frac{b}{\tilde{a}}$



stability together with the presence of the last correction term in (49), which allows price changes to become negative.[31]

To gain further intuition as to why PET traders' beliefs are eventually disappointed, notice that the intercept term in (49) is coming from uninformed traders' misunderstanding of the part of the price change due to changes in confidence alone. Following a positive displacement shock, PET agents mistakenly think that informed traders are *more optimistic* than uninformed traders. Fixing individual beliefs, as informed traders regain their edge over time, PET traders think that the average belief becomes more optimistic ($\Delta\tilde{a}_t\tilde{\mathbb{E}}_{I,t}[D_T] + \Delta\tilde{b}_t\bar{D} > 0$), and that this pushes prices up further. In reality informed traders are *less optimistic* than uninformed traders, so that, as informed traders regain their edge, the average belief actually becomes less optimistic over time and closer to

[31]Internet Appendix B.3 provides additional details of how reversals may only occur once $\frac{b_t}{\tilde{a}_t} < 1$.

Electronic copy available at: https://ssrn.com/abstract=4666338

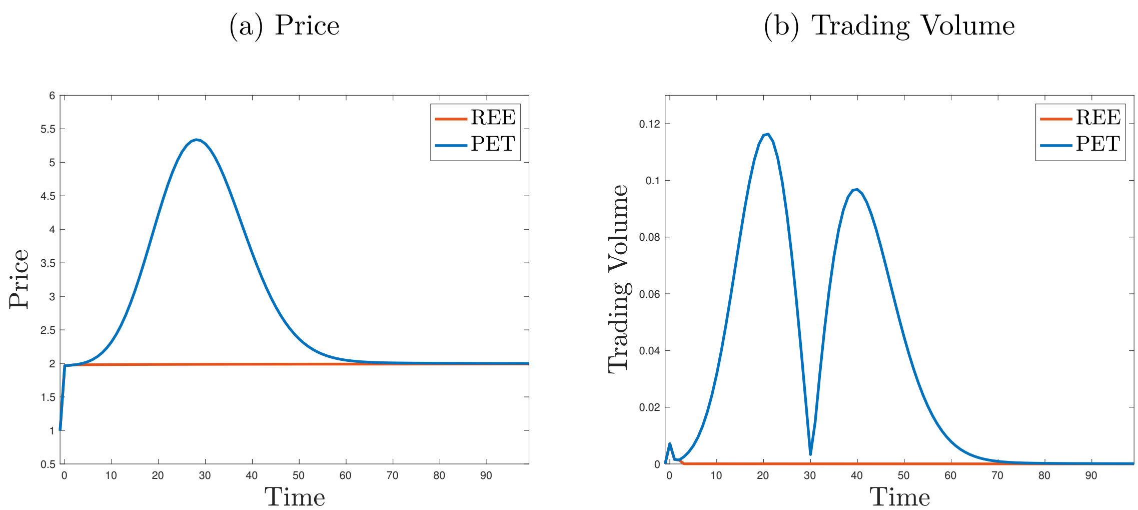
the rational benchmark ($\Delta a_t \mathbb{E}_{I,t}[D_T] + \Delta b_t \mathbb{E}_{U,t} < 0$). This puts a negative (corrective) pressure on prices. By over-estimating the part of the price change due to changes in confidence levels, partial equilibrium thinkers eventually expect price rises that are higher than the price changes that they observe. When this occurs, their beliefs are disappointed, leading them to become more pessimistic, and the bubble to burst.

Figure 2 shows the path of equilibrium outcomes following a displacement shock. Initially, as the economy enters the unstable region, prices and beliefs accelerate away from fundamentals in a convex way, and reach levels several multiples of the fundamental value of the asset (Greenwood et al. 2019). As the strength of the feedback effect weakens, and the economy re-enters the stable region, PET agents' expectations are disappointed, leading the bubble to burst, and prices and beliefs to converge back towards fundamentals. Partial equilibrium thinking naturally delivers these key characteristics of bubbles by exploiting the properties of unstable regions. The duration of the bubble is then longer and its amplitude greater when the uncertainty associated with the displacement is higher, and it takes longer to resolve over time, as in these cases equilibrium dynamics spend longer in the non-stationary region. Therefore, the exact shape of the bubble depends on the speed with which information about the displacement becomes available over time. If information about the displacement is revealed slowly at first, and at a faster rate once the bubble bursts, the model can deliver a slower boom and a faster crash (Ordonez 2013).

Finally, while the initial stage of the bubble is associated with high trading volume (Barberis 2018, Hong and Stein 2007), our model is also consistent with recent evidence in DeFusco et al. (2020) that documents a quiet period before the bust, during which trading volume is falling while prices are still rising. Partial equilibrium thinking leads to endogenously heterogeneous beliefs, and during the formation of the bubble disagreement increases initially at an increasing and then at a decreasing rate.

Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 2: Bubbles and crashes following a displacement. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$, and we let its realized value be $\omega = \mu_0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim N(0, \tau_s^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. This figure compares the path of equilibrium prices and trading volume, under rational expectations and under partial equilibrium thinking.



2.5 Negative Bubbles

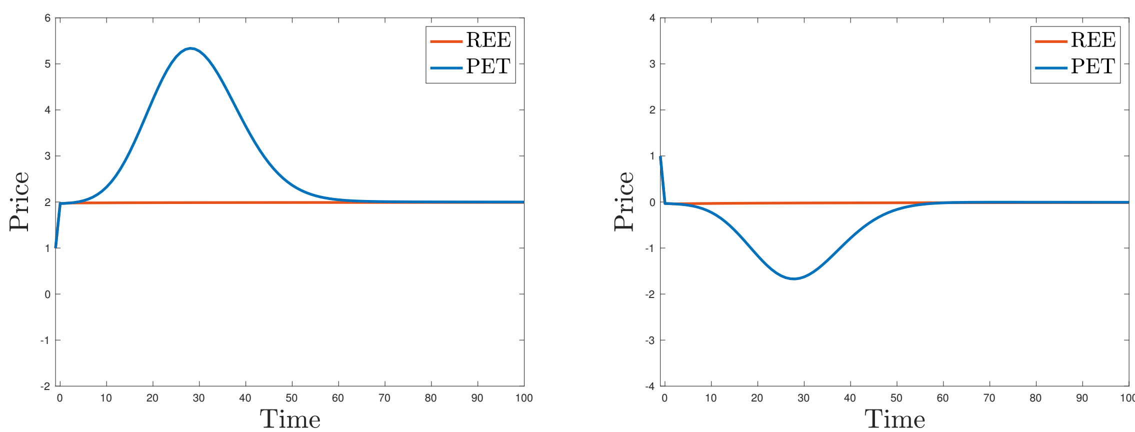
Interestingly, negative bubbles with $\mu_0 < 0$ are not merely symmetric, and instead are dampened relative to positive bubbles, as shown in Figure 3. To understand why this is the case, we ought to focus on the true and perceived risk-premium components. Regardless of the sign of the displacement shock, the gradual resolution of uncertainty over time exerts an upward force on prices, as the greater level of aggregate confidence reduces the risk-premium component. However, PET agents under-estimate this upward force, as they believe that other uninformed traders are not learning and becoming more confident over time. By under-estimating the increase in risk-bearing capacity, they then under-estimate the upward force on prices coming from changes in risk premia, and instead attribute part of this to better fundamentals. This force is at play both when the cash flow shock of the displacement is positive, and when it is negative, therefore amplifying positive bubbles and dampening negative ones. This in contrast to equilibrium dynamics with constant

Electronic copy available at: https://ssrn.com/abstract=4666338

price extrapolation, where this dampening channel is absent, and where negative bubbles would actually be more pronounced than positive ones following a displacement shock.[32]

Figure 3: Asymmetry between Positive and Negative Bubbles. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ with $\epsilon_t \sim N(0, \tau_s^{-1})$ each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \ \forall t > 1$. This figure compares the path of equilibrium prices for positive and negative bubbles. For a given size shock in absolute value, negative bubbles are dampened relative to positive bubbles.

(a) Positive Bubble $\mu_0 > 0$ (b) Negative Bubble $\mu_0 < 0$



2.6 Other Types of Displacements

A key lesson from our analysis so far is that shocks that generate bubbles and crashes must have two properties: they must shift the economy to an unstable region, and such a shift must be temporary. So far, we have considered one possible way to achieve this via a positive shock that creates uncertainty, which gradually resolves over time. However, the sources of variation in $\frac{b_t}{\bar{a}_t}$ discussed in Proposition 2 are informative about other types of shocks which may contribute to the formation of bubbles and crashes.

[32]Intuitively, the initial increase in uncertainty associated with the displacement exerts a downward pressure on prices, which dampens positive cash flow shocks, and amplifies negative cash flow shocks. Fixing the size of the cash flow shock in absolute value, this asymmetry then leads to a greater initial price change following a negative shock relative to the same size positive shock. Extrapolating a greater initial price change with constant price extrapolation then leads to more amplified dynamics in response to negative shocks.

Electronic copy available at: https://ssrn.com/abstract=4666338

Specifically, we can write the strength of the feedback effect as follows:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) < 1 \iff \left(\frac{\phi_t}{1-\phi_t}\frac{\tau_{I,t}}{\tau_{U,t}}\right)\left(\frac{\tilde{\phi}_t}{1-\tilde{\phi}_t}\frac{\tilde{\tau}_{I,t}}{\tilde{\tau}_{U,t}}\right) > 1 \tag{53}$$

where the second inequality simply follows from re-arranging the first one, and using the definition of the true and perceived informational edges.[33] Moreover, (53) generalizes our earlier expressions by allowing the fraction of informed agents in the market to be time-varying, and by allowing uninformed agents to be misspecified about this quantity ($\tilde{\phi}_t \neq \phi_t$). There are four components of the information structure that can then lead to time-variation in the strength of the feedback effect: the true and the perceived confidence of informed agents relative to uninformed agents, and the true and the perceived composition of agents in the market. Temporary shocks to these quantities can also contribute to the time-varying strength of the feedback effect.

For example, Greenwood and Nagel (2009) find that young inexperienced investors increased exposure to technology stocks during the dot.com bubble, and decreased it during the crash. More generally, historical narratives associate displacements with large changes in the composition of agents in the market (Brennan 2004, Aliber and Kindleberger 2015). This paper highlights how changes in the composition of traders constitute another source of time-variation in the strength of the feedback effect, and hints to how the timing of these changes can play an important role in determining the shape and amplitude of bubbles.

3 Inter-temporal Trading Motives

When explaining the stage of ‘euphoria’ characteristic of bubbles, Kindleberger (1978) describes how “[i]nvestors buy goods and securities to profit from the capital gains associ-

[33]Re-arranging the first inequality, we get: $(1+\zeta_t) > \left(\frac{1+\tilde{\zeta}_t}{\tilde{\zeta}_t}\right) \iff \zeta_t\tilde{\zeta}_t > 1$.

Electronic copy available at: https://ssrn.com/abstract=4666338

ated with the anticipated increases in the prices of these goods and securities." So far, we have been silent on the role of destabilizing speculation in contributing to the formation of bubbles, as we focused on understanding the role of higher order beliefs in *misinference* in isolation of its role in *forecasting*: while partial equilibrium thinking affects how traders interpret past outcomes, speculative motives depend on traders' beliefs of future equilibrium prices. In this respect, partial equilibrium thinking provides a micro-foundation for the existence of mispricing, while higher order beliefs in forecasting are more useful in understanding whether mispricing persists or whether it is arbitraged away (Abreu and Brunnermeier 2002).

To study how partial equilibrium thinking interacts with speculative motives, we now change agents' objective function. Instead of having agents who are only concerned with forecasting the terminal dividend as in (3), we now let agents have mean-variance utility over next period wealth, which leads them to forecast next period's payoff:[34]

$$\Pi_{t+1} = \beta P_{t+1} + (1 - \beta) D_t \tag{54}$$

which simply reflects traders' beliefs that with probability β the asset is alive next period, and is worth P_{t+1}, and with probability $(1 - \beta)$ the asset dies, and pays out a terminal dividend $D_t = \bar{D} + \sum_{j=0}^{t} u_j$ in normal times and $D_t = \bar{D} + \sum_{j=0}^{t} u_j + \omega$ following a displacement. Taking first order conditions, we have that agents now trade according to the following asset demand function, given their beliefs:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[\Pi_{t+1}]} \tag{55}$$

[34]Keeping the mean-variance utility function shuts down any potential wealth effect. See An et al. (2022) and Han and Makarov (2021) for studies focusing on this channel during bubbles. Relatedly, our uninformed traders consistently loose money but do not face margin constraints. Kogan et al. (2006) show that irrational traders can have a substantial price impact even when their wealth approaches zero. Kogan et al. (2017) provide conditions for behavioral agents to survive in a dynamic trading setting.

Electronic copy available at: https://ssrn.com/abstract=4666338

In Internet Appendix B.4 we solve the model with speculative motives using the same three steps as in Section 2, and show that the true price function is linear in agents' beliefs, and that partial equilibrium thinking still provides a micro-foundation for price extrapolation:

$$P_t = a_t \mathbb{E}_{I,t}[\Pi_{t+1}] + b_t \mathbb{E}_{I,t}[\Pi_{t+1}] - c_t \tag{56}$$

$$\mathbb{E}_{U,t}[\Pi_{t+1}] = \mathbb{E}_{U,t-1}[\Pi_{t+1}] + \theta_t \left(P_{t-1} - \tilde{P}_{t-1|t-2} \right) \tag{57}$$

where a_t, b_t, c_t and θ_t are once again constant in normal times, but become time-varying following a displacement. While these coefficients still depend on the properties of the environment, their functional form depends on agents' higher order beliefs. Specifically, since agents are forecasting future *endogenous* outcomes, they need to forecast other agents' future beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their own private information and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

In this section, we consider two cases.[35] First, we let informed agents understand uninformed agents' biased beliefs, which in turn implies that they understand that mispricing is predictable. Second, we consider the case where informed agents mistakenly believe that all other agents are rational and extract the right information from prices. We refer to the first type of speculators as being "PET-aware," and to the second type as being "PET-unaware." This lines up with the distinction in practical asset management

[35] While we only consider the case where all informed traders are either "PET-aware" or "PET-unaware" and this is common knowledge to them, Abreu and Brunnermeier (2002) provide a comprehensive study of how higher order beliefs in forecasting future outcomes can make mispricing persistent before the eventual bursting of the bubble. Our paper is complementary to theirs and our core contribution considers a very distinct channel, which is why we shut down speculative motives in the main part of our analysis: our focus is on how higher order beliefs affect inference from past outcomes and provides an explanation of why mispricing might exist in the first place, which is instead taken as given in Abreu and Brunnermeier (2002).

Electronic copy available at: https://ssrn.com/abstract=4666338

between investors who think about behavioral biases in the market, and those who only concentrate on the gap between market prices and their estimates of fundamentals.

Figure 4 contrasts the dynamics of equilibrium outcomes in normal times and following a displacement, with and without speculative motives. As in the case without speculation, panel (a) shows that normal times dynamics only exhibit a small degree of momentum and speculative motives keep prices closer to fundamentals. After a displacement shock, however, panel (b) of Figure 4 makes clear that the dynamics heavily depend on the behavior of informed speculators. When informed agents understand other agents' biases, they engage in destabilizing speculation and amplify the bubble. Intuitively, when informed agents realize that mispricing is predictable, they understand that higher prices today translate into more optimistic beliefs by uninformed agents and higher prices tomorrow. This increases informed agents' expected capital gains and induces them to demand more of the asset today, inflating prices further (as in De Long et al. 1990). At some point the extrapolation of uninformed agent runs out of steam as PET traders' beliefs are disappointed (by the same mechanism described in Section 2.4). When this is the case, Informed speculators realize that prices will start falling, and thus start speculating in the opposite direction, amplifying the crash. In other words, informed speculators realize that a lower price fall during the burst will translate into more pessimistic beliefs for uninformed traders. This increases the incentives for informed speculators to short the asset, leading to a further price fall.

These results are also consistent with those we obtain in Internet Appendix D, where we allow informed traders to maximize utility over terminal wealth (as opposed to next period wealth), as in He and Wang (1995). Even in that case, dynamic trading motives generate a two-way feedback effect between prices and expected capital gains, and this further amplifies the two-way feedback effect between prices and beliefs due to misinference.

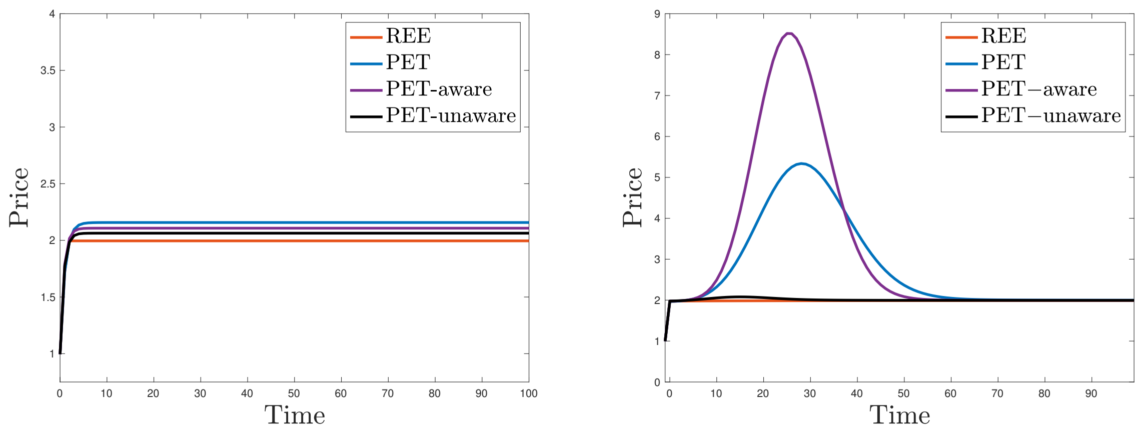
To take advantage of predictable mispricing, "PET-aware" speculators require a high

Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 4: Normal Times and Bubbles and crashes with speculators. Panel (a) compares the path of equilibrium prices under rational expectations, partial equilibrium thinking, "PET-aware" speculation, and "PET-unaware" speculation in normal times. Starting from a normal times steady state, Panel (b) considers a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ in each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0 \ \forall t > 1$.

(a) Normal times

(b) Displacement shock



level of understanding of other agents' actions and beliefs. Alternatively, we consider the case where informed agents mistakenly believe that they live in a rational world and think that uninformed agents are able to recover the right information from past prices. In this case, informed agents believe that any mispricing will be corrected next period. This leads them to trade more aggressively on their own information, thus keeping prices closer to fundamentals, and arbitraging the bubble away.

4 Conclusion

This paper makes two contributions. First, we provide a micro-foundation for the degree of price extrapolation with a dynamic theory of "Partial Equilibrium Thinking" (PET), in which uninformed agents mistakenly attribute any price change they observe to new information alone, when in reality part of the price change is due to other agents' buying/selling pressure (Bastianello and Fontanier 2023). We show that when agents think

Electronic copy available at: https://ssrn.com/abstract=4666338

in partial equilibrium the degree of extrapolation varies with the information structure, and is decreasing in informed agents' informational edge.

Second, we draw a distinction between normal times shocks which do not lead to large swings in the aggregate edge of informed and uninformed traders, and "displacement shocks," which instead do. Consistent with the Kindleberger (1978) narrative of bubbles, not every large upswing leads inevitably to a crash (Greenwood et al. 2019). Instead, bubble and crashes only occur following displacement type of shocks.

Specifically, we show that in normal times, informed agents' edge is constant, and PET delivers constant and weak price extrapolation that generates momentum. By contrast, following a displacement, informed agents' edge is temporarily wiped out, and PET agents' degree of extrapolation is stronger at first, but then gradually dies down, leading to bubbles and endogenous crashes. This provides a unifying theory of both weak departures from rationality in normal times, and of extreme bubbles and crashes following a displacement.

Studying the empirical properties of the time-variation in price extrapolation is an important avenue for future research (Cassella and Gulen 2018, Bybee 2023). Not only would this shed further light on traders' expectation formation process, but it also has important policy implications. For example, our work suggests that policymakers should be weary of creating conditions that decrease the aggregate edge of informed traders, such a long periods of cheap credit that may increase the proportion of retail traders. Additionally, whether transaction taxes or other financial regulation tools could prevent run-ups in extrapolation is an interesting question. Finally, our micro-foundation for the degree of extrapolation could help us further our understanding of whether monetary policy can be used to lean against the wind and prevent the formation of bubbles. Our analysis suggests that extrapolative beliefs can be relatively innocuous in normal times, but that an increasing degree of extrapolation is a sign of bubble formation. How traders would adjust their learning heuristics in the face of an increase in interest rates, however,

Electronic copy available at: https://ssrn.com/abstract=4666338

remains an open question.

References

Abreu, D. and Brunnermeier, M. K. (2002), 'Synchronization risk and delayed arbitrage', *Journal of Financial Economics* **66**(2-3), 341–360.

Adam, K. and Marcet, A. (2011), 'Internal rationality, imperfect market knowledge and asset prices', *Journal of Economic Theory* **146**(3), 1224–1252.

Adam, K., Marcet, A. and Beutel, J. (2017), 'Stock price booms and expected capital gains', *American Economic Review* **107**(8), 2352–2408.

Aliber, R. Z. and Kindleberger, C. P. (2015), *Manias, panics, and crashes: A history of financial crises*, Springer.

An, L., Lou, D. and Shi, D. (2022), 'Wealth redistribution in bubbles and crashes', *Journal of Monetary Economics* **126**, 134–153.

Andrade, S. C., Bian, J. and Burch, T. R. (2013), 'Analyst coverage, information, and bubbles', *Journal of Financial and Quantitative Analysis* **48**(5), 1573–1605.

Andreassen, P. B. and Kraus, S. J. (1990), 'Judgmental extrapolation and the salience of change', *Journal of Forecasting* **9**(4), 347–372.

Bae, K.-H., Stulz, R. M. and Tan, H. (2008), 'Do local analysts know more? a cross-country study of the performance of local analysts and foreign analysts', *Journal of Financial Economics* **88**(3), 581–606.

Bagehot, W. (1873), *Lombard Street: A description of the money market*, Wiley, New York.

Barberis, N. (2018), Psychology-based models of asset prices and trading volume, *in* 'Handbook of Behavioral Economics: Applications and Foundations 1', Vol. 1, Elsevier, pp. 79–175.

Barberis, N., Greenwood, R., Jin, L. and Shleifer, A. (2018), 'Extrapolation and bubbles', *Journal of Financial Economics* **129**(2), 203–227.

Bastianello, F. and Fontanier, P. (2023), Expectations and learning from prices, Working paper.

Blanchard, O. J. (1985), 'Debt, deficits, and finite horizons', *Journal of Political Economy*

Electronic copy available at: https://ssrn.com/abstract=4666338

93(2), 223–247.

Bohren, J. A. (2016), 'Informational herding with model misspecification', *Journal of Economic Theory* **163**, 222–247.

Bohren, J. A. and Hauser, D. N. (2021), 'Learning with heterogeneous misspecified models: Characterization and robustness', *Econometrica* **89**(6), 3025–3077.

Bordalo, P., Gennaioli, N., Kwon, S. Y. and Shleifer, A. (2021), 'Diagnostic bubbles', *Journal of Financial Economics* **141**(3), 1060–1077.

Branch, W. A. and Evans, G. W. (2011), 'Learning about risk and return: A simple model of bubbles and crashes', *American Economic Journal: Macroeconomics* **3**(3), 159–191.

Brennan, M. J. (2004), 'How did it happen?', *Economic Notes* **33**(1), 3–22.

Brunnermeier, M. K. and Nagel, S. (2004), 'Hedge funds and the technology bubble', *Journal of Finance* **59**(5), 2013–2040.

Brunnermeier, M. K. and Oehmke, M. (2013), 'Bubbles, financial crises, and systemic risk', *Handbook of the Economics of Finance* **2**, 1221–1288.

Bybee, J. L. (2023), 'The ghost in the machine: Generating beliefs with large language models'.

Case, K. E., Shiller, R. J. and Thompson, A. (2012), 'What have they been thinking? home buyer behavior in hot and cold markets', *Brookings Papers and Proceedings* **45**, 265–315.

Cassella, S. and Gulen, H. (2018), 'Extrapolation bias and the predictability of stock returns by price-scaled variables', *Review of Financial Studies* **31**(11), 4345–4397.

Crawford, V. P., Costa-Gomes, M. A. and Iriberri, N. (2013), 'Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications', *Journal of Economic Literature* **51**(1), 5–62.

Da, Z., Huang, X. and Jin, L. J. (2021), 'Extrapolative beliefs in the cross-section: What can we learn from the crowds?', *Journal of Financial Economics* **140**(1), 175–196.

De Long, J. B., Shleifer, A., Summers, L. H. and Waldmann, R. J. (1990), 'Positive feedback investment strategies and destabilizing rational speculation', *Journal of Finance* **45**(2), 379–395.

DeFusco, A. A., Nathanson, C. G. and Zwick, E. (2020), Speculative dynamics of prices and

Electronic copy available at: https://ssrn.com/abstract=4666338

volume, Working paper, National Bureau of Economic Research.

DeMarzo, P., Kaniel, R. and Kremer, I. (2007), 'Technological innovation and real investment booms and busts', *Journal of Financial Economics* **85**(3), 735–754.

DeMarzo, P. M., Vayanos, D. and Zwiebel, J. (2003), 'Persuasion bias, social influence, and unidimensional opinions', *Quarterly Journal of Economics* **118**(3), 909–968.

Diamond, D. W. and Verrecchia, R. E. (1981), 'Information aggregation in a noisy rational expectations economy', *Journal of Financial Economics* **9**(3), 221–235.

Enke, B. and Zimmermann, F. (2019), 'Correlation neglect in belief formation', *The Review of Economic Studies* **86**(1), 313–332.

Esponda, I. and Pouzo, D. (2016), 'Berk–nash equilibrium: A framework for modeling agents with misspecified models', *Econometrica* **84**(3), 1093–1130.

Evans, G. W. and Honkapohja, S. (1999), 'Learning dynamics', *Handbook of Macroeconomics* **1**, 449–542.

Eyster, E. and Rabin, M. (2005), 'Cursed equilibrium', *Econometrica* **73**(5), 1623–1672.

Eyster, E. and Rabin, M. (2010), 'Naive herding in rich-information settings', *American Economic Journal: Microeconomics* **2**(4), 221–43.

Eyster, E., Rabin, M. and Weizsäcker, G. (2018), An experiment on social mislearning, Rationality and Competition Discussion Paper Series 73.

Frick, M., Iijima, R. and Ishii, Y. (2020), 'Misinterpreting others and the fragility of social learning', *Econometrica* **88**(6), 2281–2328.

Fudenberg, D., Romanyuk, G. and Strack, P. (2017), 'Active learning with a misspecified prior', *Theoretical Economics* **12**(3), 1155–1189.

Fuster, A., Hebert, B. and Laibson, D. (2012), 'Natural expectations, macroeconomic dynamics, and asset pricing', *NBER Macroeconomics Annual* **26**(1), 1–48.

Gagnon-Bartsch, T. and Rabin, M. (2016), Naive social learning, mislearning, and unlearning, Working paper, Harvard University.

Galbraith, J. K. (1954), *The Great Crash: 1929*, Houghton Mifflin Harcourt.

Giglio, S., Maggiori, M. and Stroebel, J. (2016), 'No-bubble condition: Model-free tests in

Electronic copy available at: https://ssrn.com/abstract=4666338

housing markets', *Econometrica* **84**(3), 1047–1091.

Glaeser, E. L. and Nathanson, C. G. (2017), 'An extrapolative model of house price dynamics', *Journal of Financial Economics* **126**(1), 147–170.

Gompers, P. A. and Metrick, A. (2001), 'Institutional investors and equity prices', *The Quarterly Journal of Economics* **116**(1), 229–259.

Greenwood, R. and Hanson, S. G. (2015), 'Waves in ship prices and investment', *Quarterly Journal of Economics* **130**(1), 55–109.

Greenwood, R., Hanson, S. G., Shleifer, A. and Sørensen, J. A. (2022), 'Predictable financial crises', *The Journal of Finance* **77**(2), 863–921.

Greenwood, R. and Nagel, S. (2009), 'Inexperienced investors and bubbles', *Journal of Financial Economics* **93**(2), 239–258.

Greenwood, R. and Shleifer, A. (2014), 'Expectations of returns and expected returns', *Review of Financial Studies* **27**(3), 714–746.

Greenwood, R., Shleifer, A. and You, Y. (2019), 'Bubbles for Fama', *Journal of Financial Economics* **131**(1), 20–43.

Grossman, S. (1976), 'On the efficiency of competitive stock markets where trades have diverse information', *Journal of Finance* **31**(2), 573–585.

Han, B. Y. and Makarov, I. (2021), Feedback trading and bubbles, Working paper.

Harrison, J. M. and Kreps, D. M. (1978), 'Speculative investor behavior in a stock market with heterogeneous expectations', *Quarterly Journal of Economics* **92**(2), 323–336.

He, H. and Wang, J. (1995), 'Differential information and dynamic behavior of stock trading volume', *The Review of Financial Studies* **8**(4), 919–972.

Hirshleifer, D. (2015), 'Behavioral finance', *Annual Review of Financial Economics* **7**, 133–159.

Hong, H., Lim, T. and Stein, J. C. (2000), 'Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies', *The Journal of Finance* **55**(1), 265–295.

Hong, H. and Stein, J. C. (1999), 'A unified theory of underreaction, momentum trading, and overreaction in asset markets', *Journal of Finance* **54**(6), 2143–2184.

Hong, H. and Stein, J. C. (2007), 'Disagreement and the stock market', *Journal of Economic*

Electronic copy available at: https://ssrn.com/abstract=4666338

Perspectives **21**(2), 109–128.

Jin, L. J. and Peng, C. (2023), Law of small numbers in financial markets: Theory and evidence, Working paper.

Jin, L. J. and Sui, P. (2022), 'Asset pricing with return extrapolation', *Journal of Financial Economics* **145**(2), 273–295.

Jordà, Ò., Schularick, M. and Taylor, A. M. (2015), 'Leveraged bubbles', *Journal of Monetary Economics* **76**, S1–S20.

Keynes, J. M. (1936), *The general theory of employment, interest, and money*, Macmillan.

Kindleberger, C. P. (1978), *Manias, panics, and crashes; a history of financial crises*, Basic Books, New York.

Kogan, L., Ross, S. A., Wang, J. and Westerfield, M. M. (2006), 'The price impact and survival of irrational traders', *The Journal of Finance* **61**(1), 195–229.

Kogan, L., Ross, S. A., Wang, J. and Westerfield, M. M. (2017), 'Market selection', *Journal of Economic Theory* **168**, 209–236.

Kogan, S., Makarov, I., Niessner, M. and Schoar, A. (2023), 'Are cryptos different? evidence from retail trading', *NBER Working Paper* .

Kübler, D. and Weizsäcker, G. (2004), 'Limited depth of reasoning and failure of cascade formation in the laboratory', *Review of Economic Studies* **71**(2), 425–441.

Laarits, T. and Sammon, M. (2022), 'The retail habitat', *Available at SSRN 4262861* .

Liao, J., Peng, C. and Zhu, N. (2021), 'Extrapolative bubbles and trading volume', *Review of Financial Studies* **35**(4), 1682–1722.

Malmendier, U. and Nagel, S. (2011), 'Depression babies: do macroeconomic experiences affect risk taking?', *The Quarterly Journal of Economics* **126**(1), 373–416.

Marcet, A. and Sargent, T. J. (1989), 'Convergence of least squares learning mechanisms in self-referential linear stochastic models', *Journal of Economic theory* **48**(2), 337–368.

Ordonez, G. (2013), 'The asymmetric effects of financial frictions', *Journal of Political Economy* **121**(5), 844–895.

Pástor, L. and Veronesi, P. (2009), 'Technological revolutions and stock prices', *American Eco-*

Electronic copy available at: https://ssrn.com/abstract=4666338

nomic Review **99**(4), 1451–83.

Penczynski, S. P. (2017), 'The nature of social learning: Experimental evidence', *European Economic Review* **94**, 148–165.

Scheinkman, J. A. and Xiong, W. (2003), 'Overconfidence and speculative bubbles', *Journal of Political Economy* **111**(6), 1183–1220.

Schmidt-Engelbertz, P. and Vasudevan, K. (2023), 'Higher order beliefs in financial markets', *Working Paper* .

Shiller, R. J. (2000), *Irrational Exuberance*, Princeton University Press, Princeton.

Smith, V. L., Suchanek, G. L. and Williams, A. W. (1988), 'Bubbles, crashes, and endogenous expectations in experimental spot asset markets', *Econometrica* pp. 1119–1151.

Soros, G. (2015), *The Alchemy of Finance*, John Wiley & Sons.

Svenson, O. (1981), 'Are we all less risky and more skillful than our fellow drivers?', *Acta Psychologica* **47**(2), 143–148.

Tirole, J. (1985), 'Asset bubbles and overlapping generations', *Econometrica* **53**(6), 1499–1528.

Veldkamp, L. L. (2023), *Information choice in macroeconomics and finance*, Princeton University Press.

Xiong, W. (2013), Bubbles, crises, and heterogeneous beliefs, Working paper, National Bureau of Economic Research.

Yan, X. and Zhang, Z. (2009), 'Institutional investors and equity returns: are short-term institutions better informed?', *The Review of Financial Studies* **22**(2), 893–924.

A Proofs

A.1 Proof of Proposition 1: Micro-foundation

Combining (6) and (21) , we find that:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \left(\frac{1}{\tilde{a}}\right) \Delta P_{t-1} \tag{A.1}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

which provides a micro-foundation for extrapolative beliefs.

To see how the size of the bias varies with informed traders' edge, start from (24):

$$\tilde{u}_{t-1} = u_{t-1} + \left(\frac{b}{\tilde{a}}\right)\tilde{u}_{t-2} \tag{A.2}$$

If we consider the impulse response function to a one-off shock to fundamentals in period $t = 1$, so that $u_t \neq 0$ for $t = 1$ and $u_t = 0$ for $t > 1$, we can iterate the above expression backwards, and find that:

$$\tilde{u}_t = \left(\frac{b}{\tilde{a}}\right)^{t-1} u_1 \tag{A.3}$$

which shows that while uninformed traders extract the right signal in the first period after the shock, they extract a biased signal in each period thereafter. Specifically, since $u_t = 0$ for $t > 1$, we have that:

$$\tilde{u}_t - u_t = \left(\frac{b}{\tilde{a}}\right)^{t-1} u_1 \tag{A.4}$$

so that for a given fundamental shock u_1 the bias is increasing in the strength of the feedback effect $b/\tilde{a}$.Since the strength of the feedback effect in (28) is decreasing in the true and perceived informational edges, it follows that the bias in uninformed traders' beliefs is also decreasing in both these terms. □

A.2 Proof of Proposition 2: Strength of the Feedback Effect

Combining (8) with (28), we find that:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) = \left(\frac{1}{1+\frac{1}{\frac{1}{\phi}-1}\frac{\tau_I}{\tau_U}}\right)\left(1+\left(\frac{1}{\phi}-1\right)\frac{1}{\frac{\tilde{\tau}_I}{\tilde{\tau}_U}}\right) \tag{A.5}$$

The first equality shows that the strength of the feedback effect is decreasing in both the true informational edge, ζ, and in uninformed agents' perception of it, $\tilde{\zeta}$. The second

Electronic copy available at: https://ssrn.com/abstract=4666338

equality shows that the strength of the feedback effect is decreasing in the fraction of informed agents in the market, ϕ, and in the true and perceived confidence of informed agents relative to uninformed agents ${}^{\tau_I}/_{\tau_U}$, ${}^{\tilde{\tau}_I}/_{\tilde{\tau}_U}$. $\square$

A.3 Proof of Proposition 3: Deviations from Rationality

When traders have rational expectations, they infer the right information from prices at each point in time. Following a one-off shock in period 0, $\mathbb{E}_{U,t}[D_T]^{REE} = \bar{D} + u_0$ for $t > 0$. This reflects that rational uninformed traders understand that there is no new information after period 0, and that all other price changes they observe are due to the lagged response of all uninformed traders who are also learning information from prices. Following the second price rise, they no longer update their beliefs. The corresponding equilibrium prices are then given by:

$$P_t^{REE} = \bar{P} + \Delta P_0 + \Delta P_1 + \underbrace{\sum_{j=2}^{t} \Delta P_t}_{=0} = \bar{P} + au_0 + bu_0 \quad \forall t > 0 \tag{A.6}$$

where $\sum_{j=2}^{t} \Delta P_t = 0$ as neither informed nor uninformed agents update their beliefs after period $t = 1$, and in normal times the risk-premium component $\left(\frac{\mathcal{A}Z}{\phi\tau_I + (1-\phi)\tau_U}\right)$ is also constant over time.

On the other hand, from (11) and (A.1), together with the fact that in normal times $a = \tilde{a}$, we know that when uninformed traders think in partial equilibrium, equilibrium beliefs and prices are given by:

$$\mathbb{E}_{U,t}[D_T] = \bar{D} + u_0 + \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \quad \forall t > 1 \tag{A.7}$$

$$P_t = \bar{P} + au_0 + bu_0 + \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) \quad \forall t > 1 \tag{A.8}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

Comparing PET to REE outcomes, we see that when traders think in partial equilibrium, deviations from rational outcomes are given by:

$$\mathbb{E}_{U,t}[D_T] - \mathbb{E}_{U,t}^{REE}[D_T] = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j u_0 \quad \forall t > 1 \tag{A.9}$$

$$P_t - P_t^{REE} = \sum_{j=2}^{t} \left(\frac{b}{\tilde{a}}\right)^j (au_0) = \sum_{j=1}^{t-1} \left(\frac{b}{\tilde{a}}\right)^j (bu_0) \quad \forall t > 1 \tag{A.10}$$

where the last equality uses the fact that in normal times $\tilde{a} = a$.

From Proposition 2, we know that $\frac{b}{\tilde{a}}$ is decreasing in ζ, $\tilde{\zeta}$, ϕ, $\frac{\tau_I}{\tau_U}$ and $\frac{\tilde{\tau}_I}{\tilde{\tau}_U}$. Moreover, from (10) we know that b is also decreasing in ζ, which is itself increasing in ϕ and $\frac{\tau_I}{\tau_U}$. Combining these results with (A.9) and (A.10), we obtain the comparative statics in Proposition 3 $\forall t > 1$. In particular, when the equilibrium is stable these comparative statics also hold in $\lim_{t\to\infty}$, as the economy approaches the new steady state. □

A.4 Proof of Proposition 4: Time-varying Extrapolation

Before the displacement is announced, the degree of extrapolation in normal times is:

$$\theta = 1 + \frac{1}{\tilde{\zeta}} = 1 + \left(\frac{1}{\phi} - 1\right) \frac{\mathbb{V}_I}{\mathbb{V}_U} \tag{A.11}$$

Following a displacement, inverting equation (45) yields:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \frac{1}{\tilde{a}_{t-1}} \left(\Delta P_{t-1} - \left(\tilde{P}_{t-1|t-2} - P_{t-2}\right)\right) \tag{A.12}$$

Using the fact that $\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \tilde{u}_t + \tilde{w}_t$, and also that $\Delta P_{t-1} - \tilde{P}_{t-1|t-2} + P_{t-2} = P_{t-1} - \tilde{P}_{t-1|t-2}$, we get:

$$\mathbb{E}_{U,t}[D_T] = \mathbb{E}_{U,t-1}[D_T] + \frac{1}{\tilde{a}_{t-1}} \left(P_{t-1} - \tilde{P}_{t-1|t-2}\right) \tag{A.13}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

where $\frac{1}{\tilde{a}_{t-1}} = 1 + \frac{1}{\tilde{\zeta}_{t-1}}$ is time-varying (as discussed in the main text), and captures the strength with which partial equilibrium thinkers extrapolate price changes. □

A.5 Proof of Proposition 5: Time-varying Feedback Effect

In (51), we showed that, following a displacement, the strength of the feedback effect takes the following form:

$$\frac{b_{t-1}}{\tilde{a}_{t-1}} = \left(\frac{1}{1+\zeta_{t-1}}\right)\left(1 + \frac{1}{\tilde{\zeta}_{t-1}}\right) \tag{A.14}$$

which directly shows that the feedback effect is decreasing in both the true and perceived informational edges. The true and perceived informational edges were derived in (35) and (43) as follows:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}}\right) \tag{A.15}$$

$$\tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + ((t-1)\tau_s + \tau_0)^{-1}}\right) \tag{A.16}$$

Since both these quantities are time-varying, it follows that (A.14) is also time-varying. Taking the limit of this expression, we find that:

$$\lim_{t\to\infty} \zeta_t = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_I}\right) \tag{A.17}$$

$$\lim_{t\to\infty} \tilde{\zeta}_{t-1} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I}\right) \tag{A.18}$$

and hence that $\lim_{t\to\infty} \zeta_t < \lim_{t\to\infty} \tilde{\zeta}_{t-1}$ which directly implies $\lim_{t\to\infty} \frac{b_{t-1}}{\tilde{a}_{t-1}} < 1$.

Electronic copy available at: https://ssrn.com/abstract=4666338

Internet Appendix

B Additional Derivations

B.1 Rational Expectations

When uninformed traders have rational expectations, they perfectly understand what generates price changes they observe. In turn, this requires them to understand other traders' beliefs, and actions.

Formally, rational agents think that in period $t-1$ informed agents update their beliefs with the new fundamental information they receive, $\tilde{u}_{t-1}$:[36]

$$\tilde{\mathbb{E}}_{I,t-1}[D_T] = \tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1} \tag{B.1}$$

$$\tilde{\mathbb{V}}_{I,t-1}[D_T] = \left(\frac{\beta^2}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_I = \mathbb{V}_I \tag{B.2}$$

Moreover, they also understand that all other uninformed agents learn information from past prices. Specifically, they know that in period $t-1$ uninformed traders update their beliefs by $\tilde{u}_{t-2}$, which is the same signal that they extract from P_{t-2}:

$$\tilde{\mathbb{E}}_{U,t-1}[D_T] = \tilde{\mathbb{E}}_{U,t-2}[D_T] + \tilde{u}_{t-2} \tag{B.3}$$

$$\tilde{\mathbb{V}}_{U,t-1}[D_T] = \left(\frac{1}{1-\beta^2}\right)\sigma_u^2 \equiv \tilde{\mathbb{V}}_U = \mathbb{V}_U \tag{B.4}$$

To be clear on notation, notice that, while $\tilde{u}_{t-2}$ is in uninformed traders' information set starting in period $t-1$, $\tilde{u}_{t-1}$ is the signal that uninformed traders are extracting from prices in period t.

[36]The use of $t-1$ subscripts instead of t is to highlight that uninformed agents learn information from past prices, so that in period t they must understand what generated the price in period $t-1$, as this is the price they are extracting new information from.

Electronic copy available at: https://ssrn.com/abstract=4666338

Rational agents then think that the equilibrium price in period $t-1$ is given by:

$$P_{t-1} = \tilde{a}\tilde{\mathbb{E}}_{I,t-1}[D_T] + \tilde{b}\tilde{E}_{U,t-1}[D_T] - \tilde{c} \tag{B.5}$$

where: $\tilde{a} \equiv \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U} = \frac{\tilde{\zeta}}{1+\tilde{\zeta}}$, $\tilde{b} \equiv \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U} = \frac{1}{1+\tilde{\zeta}}$ and $\tilde{c} \equiv \frac{\mathcal{A}Z}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$. Since we saw in (B.2) and (B.4) that uninformed traders have correct beliefs about the posterior variances of both informed and uninformed traders, it follows that $\tilde{a} = a$, $\tilde{b} = b$ and $\tilde{c} = c$, where a, b and c are the coefficients in the true price function in (9).

Taking first differences of (B.2) and (B.4), substituting them into the first difference of (B.5), and using the fact that $\tilde{a} = a$, $\tilde{b} = b$ and $\tilde{c} = c$, we find that rational traders understand that price changes reflect two sources of price variation, which capture changes in beliefs of both informed and uninformed traders:

$$\Delta P_{t-1} = \underbrace{a\,\tilde{u}_{t-1}}_{\text{instantaneous response}} + \underbrace{b\,\tilde{u}_{t-2}}_{\text{lagged response}} \tag{B.6}$$

They then invert this mapping to extract the following signal from past prices:

$$\tilde{u}_{t-1} = \left(\frac{1}{a}\right)\Delta P_{t-1} - \left(\frac{b}{a}\right)\tilde{u}_{t-2} \tag{B.7}$$

Lagging the true price function (11), and substituting it into (B.7), we then find that uninformed traders are able to extract the right information from past prices:

$$\tilde{u}_{t-1} = u_{t-1} \tag{B.8}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

B.2 Displacements, Bubbles and Crashes

In normal times, the strength of the feedback effect is given by:

$$\frac{b}{\tilde{a}} = \left(\frac{1}{1+\zeta}\right)\left(1+\frac{1}{\tilde{\zeta}}\right) = \frac{1}{\zeta} < 1 \tag{B.9}$$

where the second equality follows from the fact that in normal times $\zeta = \tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I}$, and the last inequality follows from the fact that the economy must be in a stable region in normal times.

Following a displacement, the strength of the feedback effect is given by:

$$\frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_t}\right)\left(1+\frac{1}{\tilde{\zeta}_t}\right) \tag{B.10}$$

where in $t = 0$:

$$\zeta_0 = \tilde{\zeta}_0 = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (\tau_0)^{-1}} \tag{B.11}$$

and in $t > 0$:

$$\zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + ((t-1)\tau_s + \tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{B.12}$$

$$\tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I + (t\tau_s + \tau_0)^{-1}} \tag{B.13}$$

Combining (B.10) and (B.11), we find that in period $t = 0$ the strength of the feedback effect is given by:

$$\frac{b_0}{\tilde{a}_0} = \frac{1}{\zeta_0} = \frac{1}{\zeta} + \left(\frac{1}{\zeta_0} - \frac{1}{\zeta}\right) = \frac{b}{\tilde{a}} + \left(\frac{1-\phi}{\phi}\right)\left(\frac{\mathbb{V}_U - \mathbb{V}_I}{\mathbb{V}_U}\right)\frac{(\tau_0)^{-1}}{\mathbb{V}_U + (\tau_0)^{-1}} \tag{B.14}$$

where the second equality simply adds and subtracts the strength of the feedback effect

Electronic copy available at: https://ssrn.com/abstract=4666338

in normal times $\frac{b}{\tilde{a}} = \frac{1}{\zeta}$, and the last equality uses $\zeta = \tilde{\zeta} = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I}$ and (B.11) above, and rearranges.

Ceteris paribus, for the strength of the feedback effect to enter the unstable region we need the uncertainty associated with the displacement $(\tau_0)^{-1}$ to be high enough:

$$\frac{b_0}{\tilde{a}_0} > 1 \iff (\tau_0)^{-1} > \frac{\left(1 - \frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)\mathbb{V}_U}{1 - \left(1 - \frac{b}{\tilde{a}}\right)\left(\frac{\phi}{1-\phi}\right)\left(\frac{\mathbb{V}_U}{\mathbb{V}_U - \mathbb{V}_I}\right)} \tag{B.15}$$

where $(1 - b/\tilde{a}) > 0$ from (B.9). In the long run, as uncertainty about the displacement is resolved:

$$\zeta_\infty \equiv \lim_{t\to\infty} \zeta_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U}{\mathbb{V}_I} = \zeta \tag{B.16}$$

$$\tilde{\zeta}_\infty \equiv \lim_{t\to\infty} \tilde{\zeta}_t = \left(\frac{\phi}{1-\phi}\right)\frac{\mathbb{V}_U + (\tau_0)^{-1}}{\mathbb{V}_I} > \tilde{\zeta} \tag{B.17}$$

Combining these expressions:

$$\lim_{t\to\infty} \frac{b_t}{\tilde{a}_t} = \left(\frac{1}{1+\zeta_\infty}\right)\left(1 + \frac{1}{\tilde{\zeta}_\infty}\right) < \frac{b}{\tilde{a}} < 1 \tag{B.18}$$

which shows that in the long run the economy always returns to a stable region, with a steady state feedback effect that is weaker than the original normal times feedback effect. In the main text we show that when the strength of the feedback effect evolves in this way, prices and beliefs are initially non-stationary and accelerate away from fundamentals in a convex way. As the feedback effect then weakens towards its new steady state level, it eventually returns into a stable region, leading uninformed agents' beliefs to be disappointed, the bubble to burst, and prices and beliefs to converge back towards fundamentals.

Electronic copy available at: https://ssrn.com/abstract=4666338

B.3 Bursting the Bubble

To see how these forces play a joint role in bursting the bubble, and how the reversal can only occur once the economy returns to a stable region, we can substitute the definitions of $(P_{t-1} - P_{t-1|t-2})$ and $(P_{t-1} - \tilde{P}_{t-1|t-2})$ into (50), to find that beliefs evolve as follows:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \left(\frac{a_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{I,t-1}[D_T] - \mathbb{E}_0[D_T]) \\ - \left(1 - \frac{b_{t-1}}{\tilde{a}_{t-1}}\right)(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T]) + \frac{1}{\tilde{a}_{t-1}}(\tilde{c}_{t-1} - c_{t-1}) \quad \text{(B.19)}$$

where $\mathbb{E}_0[D_T] = \bar{D} + \mu_0$ is agents' unconditional prior belief when the displacement is announced. For the bubble to burst, we need $\tilde{u}_{t-1} + \tilde{w}_{t-1}$ to eventually turn negative. If we consider a one-off positive shock, such that $\mathbb{E}_{I,t-1}[D_T] = \mathbb{E}_{I,1}[D_T] > \mathbb{E}_0[D_T]$ for all $t \geq 1$, equation (B.19) makes clear that as long as the economy is in a unstable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} > 1$, PET agents continue to extract positive information from prices, and therefore become increasingly optimistic.[37] In other words, when the economy is in an unstable region, the lagged response of uninformed agents always raises prices by more than what uninformed agents would expect from changes in confidence alone. On the other hand, this is no longer the case once the economy returns to a stable region and the feedback between outcomes and beliefs runs out of steam. At the peak of the bubble uninformed agents' beliefs vastly exceed fundamentals, and the term in $(\mathbb{E}_{U,t-1}[D_T] - \mathbb{E}_0[D_T])$ dominates in determining the sign of the news that uninformed agents extract from past prices in (B.19). Once the economy returns into a stable region and $\frac{b_{t-1}}{\tilde{a}_{t-1}} < 1$, PET agents expect higher price rises than the ones they observe. As their beliefs are disappointed, they become more pessimistic ($\tilde{u}_{t-1} + \tilde{w}_{t-1} < 0$) and the bubble bursts.

[37] Notice that the last term in $\tilde{c}_{t-1} - c_{t-1} > 0$, as uninformed traders under-estimate the aggregate risk bearing capacity following a displacement.

Electronic copy available at: https://ssrn.com/abstract=4666338

B.4 Speculative Motives

To model speculative motives, we let agents have the following asset demand function conditional on their beliefs:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[\Pi_{t+1}] - P_t}{\mathcal{A}\mathbb{V}_{i,t}[\Pi_{t+1}]} \tag{B.20}$$

where the expected next period payoff is given by:

$$\Pi_{t+1} \equiv \beta P_{t+1} + (1-\beta)D_t \tag{B.21}$$

and simply reflects that with probability β the asset is alive next period and worth P_{t+1}, and with probability $(1-\beta)$ the asset dies and pays out its terminal dividend D_t.

Since agents are forecasting prices, which are *endogenous* outcomes, they now need to forecast other agents' future beliefs, which requires us to specify agents' higher order beliefs. While partial equilibrium thinking helps to pin down uninformed agents' higher order beliefs (they simply assume that all agents trade on their private information alone, and that this is common knowledge), it allows for more flexibility about informed agents' higher order beliefs.

We consider two cases. In Section B.4.1 we let informed agents be "PET-aware," so that they perfectly understand uninformed agents' biased beliefs. In Section B.4.2, we consider a case where informed agents are "PET-unaware" and mistakenly believe that all other agents are rational, and that uninformed agents extract the right information from prices. This lines up with the distinction in practical asset management between investors who concentrate on the gap between market prices and their estimates of fundamentals, and those who also think about behavioral biases in the market.

Electronic copy available at: https://ssrn.com/abstract=4666338

B.4.1 "PET−aware" Speculation

In solving the model, we proceed in the same three steps we used in the baseline model. First, we solve for the true price function which generates the prices agents observe. Second, we specify the mapping that uninformed agents use to extract information from prices. Third, we solve the model forward, starting from the steady state in normal times. The one key difference to our baseline setup is that since all agents are now forecasting an endogenous outcome, we now need to solve for the first two steps by backwards induction. To do so, we use the new steady state after the uncertainty surrounding the displacement has been resolved as our terminal point.

Step 1: True Market Clearing Price Function. To determine the true market clearing condition which determines the prices agents observe, we know that in period t all informed agent trade on the whole history of signals they have received up until that date ($\{u_j\}_{j=0}^t$, $\{s_j\}_{j=1}^t$) and all uninformed agents trade on the information they have learnt from past prices.

We define $\mathcal{D}_t \equiv \bar{D} + \sum_{j=1}^t u_t$ and $\mathcal{W}_t \equiv \frac{\tau_0}{t\tau_s+\tau_0}\mu_0 + \frac{\tau_s}{t\tau_s+\tau_0}\sum_{j=1}^t \tilde{s}_t$ to be informed agents' period t belief of normal times shocks and of the displacement respectively, and $\tilde{\mathcal{D}}_t$ and $\tilde{\mathcal{W}}_t$ are uninformed agents' beliefs about these quantities.

We can then guess that the true price function takes the following form:

$$P_t = A_t(\mathcal{D}_t + \mathcal{W}_t) + B_t(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}) - K_t \tag{B.22}$$

where $\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1}$ is the information that uninformed agents extract from past prices, and A_t, B_t and K_t are time-varying and deterministic coefficients.

To verify our guess, notice that if informed agents are aware of uninformed agents'

Electronic copy available at: https://ssrn.com/abstract=4666338

bias, their beliefs about next period payoff are given by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] = (1-\beta+\beta A_{t+1}) \underbrace{(\mathcal{D}_t + \mathcal{W}_t)}_{\mathbb{E}_{I,t}[\tilde{\mathcal{D}}_{t+1}+\tilde{\mathcal{W}}_{t+1}]} + \beta B_{t+1} \underbrace{\left(\frac{P_t - \tilde{B}_t(\bar{D}+\mu_0)+\tilde{K}_t}{\tilde{A}_t}\right)}_{\mathbb{E}_{I,t}[\tilde{\mathcal{D}}_t+\tilde{\mathcal{W}}_t]} - \beta K_{t+1} \tag{B.23}$$

$$\mathbb{V}_{I,t}[\Pi_{t+1}] = \mathbb{V}_{I,t}\left[\beta A_{t+1}u_{t+1} + \beta A_{t+1}\frac{\tau_s}{(t+1)\tau_s+\tau_0}(\omega+\epsilon_{t+1}) + (1-\beta)\omega\right] \tag{B.24}$$

$$= (\beta A_{t+1})^2 \sigma_u^2 + \left(\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2 (\tau_s)^{-1} + \left(1-\beta+\beta A_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2 (t\tau_s+\tau_0)^{-1} = \mathbb{V}_{I,t} \tag{B.25}$$

where the variance term captures how the uncertain components of expected profits in equation B.21 are (i) the future dividend component u_{t+1} ; (ii) the signal informed agents receive in period $t+1$, $s_{t+1} = \omega + \epsilon_{t+1}$; and (iii) the displacement shock ω.

Turning to uninformed agents' beliefs:

$$\mathbb{E}_{U,t}[\Pi_{t+1}] = (1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1}) + \beta\tilde{B}_{t+1}(\bar{D}+\mu_0) - \beta\tilde{K}_{t+1} \tag{B.26}$$

$$\begin{aligned}\mathbb{V}_{U,t}[\Pi_{t+1}] =& \mathbb{V}_{U,t}\left[\beta\tilde{A}_{t+1}\left(u_{t+1}+u_t+\frac{2\tau_s}{(t+1)\tau_s+\tau_0}\omega+\frac{\tau_s}{(t+1)\tau_s+\tau_0}(\epsilon_{t+1}+\epsilon_t)\right)+(1-\beta)(u_t+\omega)\right]\\ =&(\beta\tilde{A}_{t+1})^2\sigma_u^2 + (1-\beta+\beta\tilde{A}_{t+1})^2\sigma_u^2\\ &+\left(1-\beta+\beta\tilde{A}_{t+1}\frac{2\tau_s}{(t+1)\tau_s}\right)^2((t-1)\tau_s+\tau_0)^{-1}\\ &+2\left(\frac{\tau_s\beta\tilde{A}_{t+1}}{(t+1)\tau_s+\tau_0}\right)^2(\tau_s)^{-1} = \mathbb{V}_{U,t}\end{aligned} \tag{B.27}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

where the first equality captures that in period t uninformed traders are uncertain about u_t, u_{t+1}, ϵ_t, ϵ_{t+1} and ω, and the last equality simply simplifies notation to highlight that $\mathbb{V}_{U,t}$ is deterministic and time-varying.

Given these beliefs, the resulting market clearing price function is given by:

$$P_t = \left(\frac{\phi \mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{I,t}[\Pi_{t+1}] + \left(\frac{(1-\phi)\mathbb{V}_{I,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}}\right)\mathbb{E}_{U,t}[\Pi_{t+1}] - \frac{\mathcal{A}Z\mathbb{V}_{I,t}\mathbb{V}_{U,t}}{\phi \mathbb{V}_{U,t} + (1-\phi)\mathbb{V}_{I,t}} \quad \text{(B.28)}$$

Since (B.23), (B.25), (B.26) and (B.27) show that $\mathbb{E}_{I,t}[\Pi_{t+1}]$ is linear in $(\mathcal{D}_t + \mathcal{W}_t)$ and $(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1})$, $\mathbb{E}_{U,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1})$, and that $\mathbb{V}[\Pi_{t+1}]$ and $\mathbb{V}[\Pi_{t+1}]$ are deterministic, we see that the true price function does indeed take the form in (B.22). Substituting (B.23), (B.25), (B.26) and (B.27) into (B.28), and matching coefficients, yields:

$$A_t = \left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta A_{t+1}) \quad \text{(B.29)}$$

$$B_t = \left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta \tilde{A}_{t+1}) \quad \text{(B.30)}$$

$$K_t = \left(\frac{\frac{\phi}{V_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)\left(\beta K_{t+1} + \beta\frac{B_{t+1}}{\tilde{A}_t}\left(-\tilde{B}_t(\bar{D}+\mu_0)+\tilde{K}_t\right)\right) + \left(\frac{\frac{1-\phi}{V_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right) + \frac{(1-\phi)}{\mathbb{V}_{U,t}}}\right)\left(-\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)+\beta\tilde{K}_{t+1}\right)$$

Electronic copy available at: https://ssrn.com/abstract=4666338

$$+\frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}}\left(1-\beta\frac{B_{t+1}}{\tilde{A}_t}\right)+\frac{(1-\phi)}{\mathbb{V}_{U,t}}} \quad \text{(B.31)}$$

These expressions give recursive equations for the coefficients which determine equilibrium prices at each point in time. To solve for this mapping, we then need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved as the end point. Specifically, the new steady state is given by:

$$A'=\left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\right)(1-\beta+\beta A') \quad \text{(B.32)}$$

$$B'=\left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\right)(1-\beta+\beta\tilde{A}') \quad \text{(B.33)}$$

$$\begin{aligned}K'=&\left(\frac{\frac{\phi}{\mathbb{V}'_I}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\right)\left(\beta K'+\beta\frac{B'}{\tilde{A}'}\left(-\tilde{B}'(\bar{D}+\mu_0)+\tilde{K}'\right)\right)\\&+\left(\frac{\frac{1-\phi}{\mathbb{V}'_U}}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\right)\left(-\beta\tilde{B}'(\bar{D}+\mu_0)+\beta\tilde{K}'\right)\\&+\frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}'_I}\left(1-\beta\frac{B'}{\tilde{A}'}\right)+\frac{1-\phi}{\mathbb{V}'_U}}\end{aligned} \quad \text{(B.34)}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are the coefficients of the mapping PET agents use to extract information from prices in the new steady state, and which we solve for in (B.46), (B.47) and (B.48) in the next section respectively. Moreover, $\mathbb{V}'_I$ and $\mathbb{V}'_U$ are the variances of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\mathbb{V}'_I=\lim_{t\to\infty}\mathbb{V}_{I,t}=(\beta A')^2\sigma_u^2 \quad \text{(B.35)}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

$$\mathbb{V}'_U = \lim_{t\to\infty} \mathbb{V}_{U,t} = (\beta\tilde{A}')^2\sigma_u^2 + (1-\beta+\beta\tilde{A}')^2\sigma_u^2 \tag{B.36}$$

Using this steady state as our end point, we can then solve for the true price function which generates the prices agents observe by backward induction.

Step 2: Mapping to Infer Information from Prices. As in the baseline model without speculation, PET agents think that in period t informed agents trade on the information they received, $\{u_j\}_{j=1}^t$, $\{s_j\}_{j=1}^t$, and that uninformed agents only trade on their prior beliefs. Therefore, we can guess that their perceived equilibrium price function takes the following form:

$$P_t = \tilde{A}_t(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t) + \tilde{B}_t(\bar{D} + \mu_0) - \tilde{K}_t \tag{B.37}$$

where $\tilde{A}_t$, $\tilde{B}_t$ and $\tilde{K}_t$ are time-varying and deterministic coefficients.

To verify that this is the price function which would arise in equilibrium if agents traded on their own private information alone, notice that, given this price function, informed agents' beliefs would take the following form:

$$\begin{aligned}\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] =& \tilde{\mathbb{E}}_{I,t}[\beta\left(\tilde{A}_{t+1}(\tilde{\mathcal{D}}_{t+1} + \tilde{\mathcal{W}}_{t+1}) + \tilde{B}_{t+1}(\bar{D}+\mu_0) - \tilde{K}_{t+1}\right) + (1-\beta)(\tilde{\mathcal{D}}_t + \tilde{\omega})] \\ =&(1-\beta+\beta\tilde{A}_{t+1})(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t) + \beta\tilde{B}_{t+1}(\bar{D}+\mu_0) - \beta\tilde{K}_{t+1}\end{aligned} \tag{B.38}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{I,t}[\Pi_{t+1}] =& \tilde{\mathbb{V}}_{I,t}\left[\beta\tilde{A}_{t+1}\tilde{u}_{t+1} + \beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)(\tilde{\omega}+\tilde{\epsilon}_{t+1}) + (1-\beta)\tilde{\omega}\right] \\ =& \left(\beta\tilde{A}_{t+1}\right)^2\sigma_u^2 + \left(\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(\tau_s)^{-1} \\ &+ \left(1-\beta+\beta\tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s+\tau_0}\right)\right)^2(t\tau_s+\tau_0)^{-1} = \tilde{\mathbb{V}}_{I,t}\end{aligned} \tag{B.39}$$

where $\tilde{\mathbb{V}}_{I,t}$ is time-varying and deterministic. Turning to PET agents' beliefs of other

Electronic copy available at: https://ssrn.com/abstract=4666338

uninformed agents' beliefs:

$$\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] = (1 - \beta + \beta \tilde{A}_{t+1} + \beta \tilde{B}_{t+1})(\bar{D} + \mu_0) - \beta \tilde{K}_{t+1} \tag{B.40}$$

$$\begin{aligned}\tilde{\mathbb{V}}_{U,t}[\Pi_{t+1}] =& \tilde{\mathbb{V}}_{I,t}\left[\beta \tilde{A}_{t+1}(\tilde{u}_{t+1} + \tilde{u}_t) + \beta \tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s + \tau_0}\right)(2\tilde{\omega} + \tilde{\epsilon}_t + \tilde{\epsilon}_{t+1}) + (1-\beta)(\tilde{u}_t + \tilde{\omega})\right] \\ &= \left(\beta \tilde{A}_{t+1}\right)^2 \sigma_u^2 + \left(1 - \beta + \beta \tilde{A}_{t+1}\right)^2 \sigma_u^2 + 2\left(\beta \tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s + \tau_0}\right)\right)^2 (\tau_s)^{-1} \\ &\quad + \left(1 - \beta + 2\beta \tilde{A}_{t+1}\left(\frac{\tau_s}{(t+1)\tau_s + \tau_0}\right)\right)^2 (\tau_0)^{-1} = \tilde{\mathbb{V}}_{U,t}\end{aligned} \tag{B.41}$$

where $\mathbb{V}_{U,t}$ is time-varying and deterministic.[38]

Given these beliefs, the resulting market clearing price function is given by:

$$\begin{aligned}P_t = &\left(\frac{\phi \tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}}\right) \tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}] \\ &+ \left(\frac{(1-\phi)\tilde{\mathbb{V}}_{I,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}}\right) \tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}] \\ &- \frac{\mathcal{A} Z \tilde{\mathbb{V}}_{I,t} \tilde{\mathbb{V}}_{U,t}}{\phi \tilde{\mathbb{V}}_{U,t} + (1-\phi)\tilde{\mathbb{V}}_{I,t}}\end{aligned} \tag{B.42}$$

Since (B.38), (B.39), (B.40) and (B.41) show that $\tilde{\mathbb{E}}_{I,t}[\Pi_{t+1}]$ is linear in $(\tilde{\mathcal{D}}_t + \tilde{\mathcal{W}}_t)$ and $(\bar{D} + \mu_0)$, that $\tilde{\mathbb{E}}_{U,t}[\Pi_{t+1}]$ is linear in $(\bar{D} + \mu_0)$ and that $\tilde{\mathbb{V}}_{I,t+1}[\Pi_{t+1}]$ and $\tilde{\mathbb{V}}_{U,t+1}[\Pi_{t+1}]$ are deterministic, we see that given PET agents' beliefs about other agents, the price function which generates the prices they observe does indeed take the form in (B.37). Substituting

[38]In solving the model we assume that partial equilibrium thinkers believe other uninformed traders think past fundamental shocks simply did not realize - since they did not receive private information about them, they think they did not happen. Our results are robust to alternative assumptions about traders' higher order beliefs. For example, we could just as easily have assumed that PET traders believe that other uninformed traders think no news ever arrives, and having them trade on fixed prior beliefs even following a displacement.

Electronic copy available at: https://ssrn.com/abstract=4666338

(B.38), (B.39), (B.40) and (B.41) into (B.42), and matching coefficients yields:

$$\tilde{A}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1}) \tag{B.43}$$

$$\tilde{B}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{B}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) (1 - \beta + \beta \tilde{A}_{t+1} + \beta \tilde{B}_{t+1}) \tag{B.44}$$

$$\tilde{K}_t = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \right) \beta \tilde{K}_{t+1} - \frac{\mathcal{A}Z}{\frac{\phi}{\tilde{\mathbb{V}}_{I,t}} + \frac{1-\phi}{\tilde{\mathbb{V}}_{U,t}}} \tag{B.45}$$

These expressions give recursive equations for the coefficients with determine equilibrium prices at each point in time. Therefore, to solve for this mapping, we need to solve the model by backward induction. We can do this by using the new steady state after the uncertainty generated by the displacement is resolved. Specifically, uninformed agents think that the new steady state has:

$$\tilde{A}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) (1 - \beta + \beta \tilde{A}') \tag{B.46}$$

$$\tilde{B}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{B}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) (1 - \beta + \beta \tilde{A}' + \beta \tilde{B}') \tag{B.47}$$

$$\tilde{K}' = \left(\frac{\frac{\phi}{\tilde{\mathbb{V}}'_I}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{K}' + \left(\frac{\frac{1-\phi}{\tilde{\mathbb{V}}'_U}}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \right) \beta \tilde{K} - \frac{\mathcal{A}Z}{\frac{\phi}{\tilde{\mathbb{V}}'_I} + \frac{1-\phi}{\tilde{\mathbb{V}}'_U}} \tag{B.48}$$

where $\tilde{A}'$, $\tilde{B}'$ and $\tilde{K}'$ are PET agents' beliefs of the coefficients of the price function in the new steady state after the uncertainty associated with the displacement is resolved, and $\tilde{\mathbb{V}}'_I$ and $\tilde{\mathbb{V}}'_U$ are PET agents' beliefs of the variance of informed and uninformed agents in the new steady state when uncertainty is resolved:

$$\tilde{V}'_I = \lim_{t \to \infty} \tilde{\mathbb{V}}_{I,t} = (\beta \tilde{A})^2 \sigma_u^2 \tag{B.49}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

$$\tilde{V}_U' = \lim_{t\to\infty} \tilde{\mathbb{V}}_{U,t} = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 + (1-\beta)^2(\tau_0)^{-1} \tag{B.50}$$

Using this steady state as our end point, we can then solve for the mapping uninformed agents use to extract information from prices by backward induction.

Given this mapping, uninformed agents extract the following information from prices:

$$\tilde{\mathcal{D}}_{t-1} + \tilde{\mathcal{W}}_{t-1} = \frac{P_{t-1} - \tilde{B}_{t-1}(\bar{D}+\mu_0) + \tilde{K}_{t-1}}{\tilde{A}_{t-1}} \tag{B.51}$$

Or, given their information set in period t, they extract the following *new information* from the unexpected price change they observe in period $t-1$:

$$\tilde{u}_{t-1} + \tilde{w}_{t-1} = \frac{1}{\tilde{A}_{t-1}}(P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]) \tag{B.52}$$

where $\tilde{w}_{t-1} = \tilde{\mathcal{W}}_{t-1} - \tilde{\mathcal{W}}_{t-2}$. This verifies our claim in the text that PET agents extrapolate unexpected price changes even when we allow for speculative motives.

Step 3: Solving the Model Recursively. We solve for the normal times steady state before the displacement is announced by solving the system of equations in (B.46), (B.47), (B.48) and (B.32), (B.33), (B.34), using the following normal times variances:

$$\tilde{\mathbb{V}}_I = (\beta\tilde{A})^2\sigma_u^2 \tag{B.53}$$

$$\tilde{\mathbb{V}}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.54}$$

$$\mathbb{V}_I = (\beta A)^2\sigma_u^2 \tag{B.55}$$

$$\mathbb{V}_U = (\beta\tilde{A})^2\sigma_u^2 + (1-\beta+\beta\tilde{A})^2\sigma_u^2 \tag{B.56}$$

Starting from the normal times steady state, we can then simulate the equilibrium path of our economy forward for a given set of signals.

Electronic copy available at: https://ssrn.com/abstract=4666338

B.4.2 "PET−unaware" Speculation - Mistakenly Rational

If informed agents are not omniscient, and instead mistakenly believe that the world is rational, and that uninformed agents are able to recover the correct information form prices, then their posterior beliefs in (B.23) should be replaced by:

$$\mathbb{E}_{I,t}[\Pi_{t+1}] \;=\; (1-\beta+\beta A_{t+1})(\mathcal{D}_t+\mathcal{W}_t)+\beta B_{t+1}(\mathcal{D}_t+\mathcal{W}_t)-\beta K_{t+1} \tag{B.57}$$

The posterior variance is identical since, as in the "PET−aware" case, Informed agents are certain about the beliefs that Uninformed agents will have next period.

Following the same steps as in Section B.4.1 above, it follows that the equilibrium price becomes:

$$P_t = A_t(\mathcal{D}_t+\mathcal{W}_t)+B_t(\tilde{\mathcal{D}}_{t-1}+\tilde{\mathcal{W}}_{t-1})-K_t \tag{B.58}$$

where:

$$A_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta A_{t+1}+\beta B_{t+1}) \tag{B.59}$$

$$B_t = \left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)(1-\beta+\beta \tilde{A}_{t+1}) \tag{B.60}$$

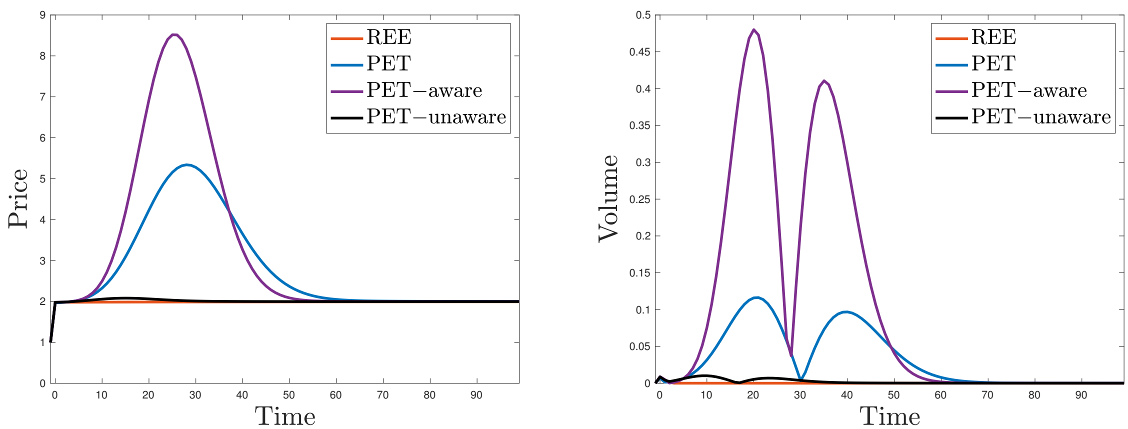
$$K_t = \left(\frac{\frac{\phi}{\mathbb{V}_{I,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)\beta K_{t+1}+\left(\frac{\frac{1-\phi}{\mathbb{V}_{U,t}}}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}}\right)\left(-\beta\tilde{B}_{t+1}(\bar{D}+\mu_0)+\tilde{K}_{t+1}\right)+\frac{\mathcal{A}Z}{\frac{\phi}{\mathbb{V}_{I,t}}+\frac{1-\phi}{\mathbb{V}_{U,t}}} \tag{B.61}$$

Since the mapping used by PET agents to extract information from prices is unchanged relative to the one in Section B.4.1, we can use this alternative price function to simulate the path of equilibrium prices and beliefs by following the same steps as in Section B.4.1. The results of these simulation for prices, beliefs, trading volume and asset demand are presented in Figure 5.

Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 5: Bubbles and crashes with “PET-aware” and “PET-unaware” speculators. Starting from a normal times steady state, a displacement $\omega \sim N(\mu_0, \tau_0^{-1})$ is announced in period $t = 0$. Informed agents then receive a signal $s_t = \omega + \epsilon_t$ in each period, where $\epsilon_1 > 0$ and $\epsilon_t = 0\ \forall t > 1$. This figure compares the path of equilibrium prices and trading volume under rational expectations, partial equilibrium thinking, “PET-aware” speculation, and “PET-unaware” speculation. “PET-aware” speculation amplifies the bubble relative to the case with no speculative motives, while “PET-unaware” speculation arbitrages the bubble away.

(a) Price (b) Trading volume



C Partially Revealing Prices

When prices are fully revealing, the extrapolation parameter used by PET agents is decreasing in informed agents’ informational edge. In this section, we study how the extrapolation parameter changes if we allow for noise, so that prices are no longer fully revealing.

C.0.1 Stochastic Supply and Information Structure

To consider the effect of noise on PET agents’ inference problem, we assume that the supply of the risky asset is stochastic, and given by $z_t \overset{iid}{\sim} N(Z, \sigma_z^2)$. To illustrate the effect of noise in the simplest possible way, we assume that agents learn the realization of the supply of the risky asset after two periods. In each period t, all agents are uncertain about $z_{t-j} \overset{iid}{\sim} N(Z, \sigma_z^2)$ for $j \leq 1$ and they know the realization of z_{t-h} for $h \geq 2$. Even

Electronic copy available at: https://ssrn.com/abstract=4666338

though one period lagged prices are partially revealing, this assumption makes prices fully revealing at further lags, thus simplifying PET agents' inference.

C.0.2 Inference Problem with Noise

When prices are fully revealing, uninformed agents think they can extract from prices the exact information that informed agents received in the previous period. This is no longer true when prices are partially revealing, as uninformed agents can only infer a noisy signal of fundamentals from prices. Specifically, in normal times, uninformed agents think that prices take the following form:

$$P_{t-1} = \tilde{a}\left(\tilde{\mathbb{E}}_{I,t-2}[D_T] + \tilde{u}_{t-1}\right) + \tilde{b}\bar{D} - \tilde{c}z_{t-1} \tag{C.1}$$

where $\tilde{a} = \frac{\phi\tilde{\tau}_I}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$, $\tilde{b} = \frac{(1-\phi)\tilde{\tau}_U}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$ and $\tilde{c} = \frac{\mathcal{A}}{\phi\tilde{\tau}_I+(1-\phi)\tilde{\tau}_U}$. Since prices are fully revealing in period $t-2$, but they are partially revealing in period $t-1$, uninformed agents extract the following noisy signal from prices:[39]

$$\frac{P_{t-1} - \tilde{a}\tilde{D}_{t-2} - \tilde{b}\bar{D} + \tilde{c}Z}{\tilde{a}} = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{C.2}$$

and we can re-write this more simply as:

$$\left(\frac{1}{\tilde{a}}\right)\left(P_{t-1} - \mathbb{E}_{t-1}[P_{t-1}]\right) = \tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}\left(z_{t-1} - Z\right) \tag{C.3}$$

This shows that uninformed agents are now uncertain as to whether the unexpected price change they observe is due to new information, or to changes in the stochastic supply of the risky asset. Either way, PET agents still extrapolate past prices to recover a (noisy) signal from them.

[39] The assumption that prices are fully revealing in period $t-2$ means that uninformed agents think they know the exact value of $\tilde{\mathbb{E}}_{I,t-2}[D_T] = \tilde{D}_{t-2}$, as opposed to being uncertain about it.

Electronic copy available at: https://ssrn.com/abstract=4666338

Given the noisy information that uninformed agents extract from prices, their beliefs in period t are given by:

$$\mathbb{E}_{U,t}[D_T] = \tilde{D}_{t-2} + \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2} \right) \left(\frac{1}{\tilde{a}}\right) (P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]) \tag{C.4}$$

$$= \tilde{D}_{t-2} + \frac{\kappa}{\tilde{a}} (P_{t-1} - \mathbb{E}_{U,t-1}[P_{t-1}]) \tag{C.5}$$

where $\kappa = \left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{\tilde{c}}{\tilde{a}}\right)^2 \sigma_z^2} \right) \leq 1$ is the weight that PET agents put on the noisy signal they extract from past prices. This shows that the extrapolation parameter θ now depends on two components:

$$\theta \equiv \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{\sigma_u^2}{\sigma_u^2 + \left(\frac{1}{\phi \tilde{\tau}_I}\right)^2 \sigma_z^2} \right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right) \frac{\tilde{\tau}_U}{\tilde{\tau}_I} \right)}_{\text{inference}} \tag{C.6}$$

where $(\tilde{\tau}_U)^{-1} = \left(\frac{1}{1-\beta^2}\right) \sigma_u^2 = (\tilde{\tau}_I)^{-1} + \sigma_u^2$ and $(\tilde{\tau}_I)^{-1} = \left(\frac{\beta^2}{1-\beta^2}\right) \sigma_u^2$. Starting from the second component in (C.6), $1/\tilde{a}$ is the extrapolation parameter that would prevail if $\sigma_z^2 = 0$ and prices were fully revealing: the more sensitive prices are to shocks, the less strongly do PET agents need to extrapolate unexpected price changes to recover the (in their mind unbiased) noisy signal $\tilde{u}_{t-1} - \frac{\tilde{c}}{\tilde{a}}(z_{t-1} - Z)$ from prices. Turning to the first component in (C.6), $\kappa \leq 1$ is the weight that PET agents put on the information they extract from prices when forming their posterior beliefs. Whenever $\sigma_z^2 > 0$, $\kappa < 1$, and PET agents extrapolate prices less strongly than when prices are fully revealing, and this simply reflects the noisy nature of the signal they are able to infer from prices.

To draw comparative statics, we can substitute the expressions for $\tilde{\tau}_I$ and $\tilde{\tau}_U$ into

Electronic copy available at: https://ssrn.com/abstract=4666338

(C.6), and re-write the extrapolation parameter in terms of the primitives of the model:

$$\theta = \frac{\kappa}{\tilde{a}} = \underbrace{\left(\frac{1}{1 + \left(\frac{1}{\phi}\right)^2 \left(\frac{\beta^2}{1-\beta^2}\right)^2 \sigma_u^2 \sigma_z^2} \right)}_{\text{weight}} \underbrace{\left(1 + \left(\frac{1-\phi}{\phi}\right) \beta^2 \right)}_{\text{inference}} \tag{C.7}$$

From this expression, we see that the extrapolation parameter is decreasing in all sources of noise (σ_u^2 and σ_z^2), as this reduces the informativeness of the signal uninformed agents extract from prices.

On the other hand, increasing the perceived information advantage ($1/\beta^2$) and the fraction of informed agents in the market (ϕ) both have two competing roles. Increasing $1/\beta^2$ (or ϕ) decreases the fully revealing extrapolation parameter $1/\tilde{a}$ as prices are more sensitive to news, but it also increases the weight κ, as prices are a more informative signal. For small enough noise, the first effect dominates, and the extrapolation parameter is decreasing in the informational edge, and in the fraction of informed agents in the market. On the other hand, if there is too much noise in prices, the second effect dominates and the comparative statics are reversed.[40]

D Dynamic Trading

D.1 Setup

In this section we consider the case where informed traders solve the full inter-temporal maximization problem, where they maximize CARA utility over terminal wealth. To do so, we make our setup as close as possible to He and Wang (1995). Our traders solve a portfolio choice problem between a risky and a riskless asset. The riskless asset is in fixed

[40] Notice that it is a more general property of learning models that the effects of learning are dampened when noise is greater. Therefore, in this section we see that in circumstances where learning is relevant, the comparative statics described in the main text still hold.

Electronic copy available at: https://ssrn.com/abstract=4666338

elastic supply, and we let the risk-free rate be zero. The risky asset is in fixed supply Z, and pays off a terminal dividend of v in period $T+1$. Turning to the information structure, a fraction ϕ of traders are informed, and receive a signal $s_t = v + \epsilon_t$ with $\epsilon_t \sim^{iid} N(0, \sigma_\epsilon^2)$. The remaining fraction $1-\phi$ of traders are instead uninformed, and learn information from past prices while engaging in partial equilibrium thinking.

In what follows, we solve the model in two ways. First, we solve the model by assuming that all traders have mean-variance utility over the fundamental value of the asset. Second, we consider the case where informed traders are sophisticated, and solve the full intertemporal maximization problem while also perfectly understanding other traders' objective function and beliefs.[41]

D.2 Mean-variance Utility

As a benchmark, we consider the case where all traders have mean-variance utility:

$$\max_{X_{i,t}} \quad \left\{ X_{i,t} \left(\mathbb{E}_{i,t}[v] - P_t \right) - \frac{1}{2} \mathcal{A} X_{i,t}^2 \mathbb{V}_{i,t}[v] \right\} \tag{D.1}$$

where $\mathcal{A}$ is the coefficient of absolute risk aversion. Traders' asset demand functions are given by:

$$X_{i,t} = \frac{\mathbb{E}_{i,t}[v] - P_t}{\mathcal{A} \mathbb{V}_{i,t}[v]} \tag{D.2}$$

As in the baseline framework, we first solve for the true price function, given agents' beliefs. Next, we solve for the price function which uninformed traders think is generating the price change they observe, both for the rational and for the PET case. Finally, we solve for equilibrium outcomes.

[41] We continue to assume that uninformed traders have mean-variance utility over the fundamental value of the asset, and believe that all other traders have mean-variance utility too. This assumption can easily be relaxed, and we maintain it here for simplicity.

Electronic copy available at: https://ssrn.com/abstract=4666338

D.2.1 True Price Function

Given the information structure, market clearing leads to the following price function:

$$P_t = \frac{\phi\tau_{I,t}}{\phi\tau_{I,t} + (1-\phi)\tau_{U,t}} E_{I,t}[v] + \frac{(1-\phi)\tau_{U,t}}{\phi\tau_{I,t} + (1-\phi)\tau_{U,t}} \mathbb{E}_{U,t}[v] - \frac{\mathcal{A}Z}{\phi\tau_{I,t} + (1-\phi)\tau_{U,t}} \tag{D.3}$$

where $\tau_{I,t} = t\tau_s + \tau_0$, $\tau_{U,t} = (t-1)\tau_s + \tau_0$, $\mathbb{E}_{I,t}[v] = \frac{\tau_s}{t\tau_s+\tau_0}\sum_{i=1}^{t} s_i + \frac{\tau_0}{t\tau_s+\tau_0}\mu_0$, and $\mathbb{E}_{U,t}[v]$ depends on the mapping uninformed traders use to extract information from prices, which we turn to next.

Before we do so, notice that we can re-write the true price function more succinctly as:

$$P_t = A_t E_{I,t}[v] + B_t \mathbb{E}_{U,t}[v] - K_t \tag{D.4}$$

where $A_t \equiv \frac{\phi\tau_{I,t}}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}$, $B_t \equiv \frac{(1-\phi)\tau_{U,t}}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}\mu_0$ and $K_t \equiv \frac{\mathcal{A}Z}{\phi\tau_{I,t}+(1-\phi)\tau_{U,t}}$.

D.2.2 Rational Mapping Used to Infer Information from Prices

When uninformed traders have rational expectations, and learn information from past prices, they are able to infer the right information, such that:

$$\mathbb{E}_{U,t}[v] = \tilde{\mathbb{E}}_{I,t-1}[v] = \mathbb{E}_{I,t-1}[v] \tag{D.5}$$

D.2.3 PET Mapping Used to Infer Information from Prices

To understand what information uninformed traders infer from prices under partial equilibrium thinking, we need to pin down uninformed traders' beliefs of what generates the price changes they observe. Specifically, PET uninformed traders think that prices evolve as follows:

$$P_t = \frac{\phi\tilde{\tau}_{I,t}}{\phi\tilde{\tau}_{I,t} + (1-\phi)\tilde{\tau}_{U,t}} \tilde{E}_{I,t}[v] + \frac{(1-\phi)\tilde{\tau}_{U,t}}{\phi\tilde{\tau}_{I,t} + (1-\phi)\tilde{\tau}_{U,t}} \mu_0 - \frac{\mathcal{A}Z}{\phi\tilde{\tau}_{I,t} + (1-\phi)\tilde{\tau}_{U,t}} \tag{D.6}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

where $\tilde{\tau}_{I,t} = t\tau_s + \tau_0$ and $\tilde{\tau}_{U,t} = \tau_0$. We can write this more succinctly as:

$$P_t = \tilde{A}_t \tilde{E}_{I,t}[v] - \tilde{K}_t \tag{D.7}$$

where $\tilde{A}_t \equiv \frac{\phi\tilde{\tau}_{I,t}}{\phi\tilde{\tau}_{I,t}+(1-\phi)\tilde{\tau}_{U,t}}$, $\tilde{K}_t \equiv -\frac{(1-\phi)\tilde{\tau}_{U,t}}{\phi\tilde{\tau}_{I,t}+(1-\phi)\tilde{\tau}_{U,t}}\mu_0 + \frac{\mathcal{A}Z}{\phi\tilde{\tau}_{I,t}+(1-\phi)\tilde{\tau}_{U,t}}$. Uninformed traders then invert this mapping to infer informed traders' previous period beliefs, which in turn pin down their own beliefs in period t:

$$\mathbb{E}_{U,t}[v] = \tilde{\mathbb{E}}_{I,t-1}[v] = \frac{1}{\tilde{A}_{t-1}} P_{t-1} + \frac{\tilde{K}_{t-1}}{\tilde{A}_{t-1}} \tag{D.8}$$

D.2.4 Results

Figure 6 plots the equilibrium price in normal times (left panel) and following a displacement (right panel) for both the rational (red line) and PET (blue line) case. We use these as benchmarks against which we can interpret the effects of adding intertemporal trading motives, which we turn to next.

D.3 Intertemporal Problem

In this section, we consider the case where informed traders have CARA utility over *terminal wealth* and perfectly understand how uninformed traders form their beliefs and trade. Moreover, we assume that uninformed traders still engage in mean-variance utility, and have the same demand function as in (D.8).[42]

We follow He and Wang (1995) as closely as possible in solving informed traders' maximization problem, and adapt their method to allow uninformed traders to engage in partial equilibrium thinking.

[42]We choose this set of assumptions because ultimately we want to understand whether allowing informed traders to have intertemporal trading motives would lead them to arbitrage the bubble away. Alternative assumptions, with greater sophistication on the part of uninformed traders, can also be accommodated.

Electronic copy available at: https://ssrn.com/abstract=4666338

We start by guessing that the price function is a linear function of traders' beliefs:

$$P_t = A_t \mathbb{E}_{I,t}[v] + B_t \mathbb{E}_{U,t}[v] - K_t = A_{P,t}\Psi_t \tag{D.9}$$

where $A_{P,t} \equiv \begin{pmatrix} -K_t & A_t & B_t \end{pmatrix}$, and $\Psi_t \equiv \begin{pmatrix} 1 & \mathbb{E}_{I,t}[v] & \mathbb{E}_{U,t}[v] \end{pmatrix}'$ is our state vector. Second, we guess that the value function takes the following form:

$$J(W_{I,t}; \Psi_t; t) = \mathbb{E}_{I,t}\left[-e^{-\mathcal{A}W_{I,T}}\right] = -e^{-\mathcal{A}W_t - \frac{1}{2}\Psi_t' U_t \Psi_t} \tag{D.10}$$

To solve for the equilibrium price function, we first need to show that Ψ_{t+1} and $Q_{t+1} \equiv P_{t+1} - P_t$ are Gaussian processes. Second, we can use CARA normal results to simplify informed traders' maximization problem given the guessed value function form at $t+1$, and find informed traders' demand function. Using the derived demand function we can then also verify by recursion that the value function takes the postulated form at t. Third, we write down uninformed traders' demand function. Fourth, we impose market clearing, and match coefficients to define $A_{P,t}$ recursively. Fifth, we solve the problem for period T, in order to start the recursion which allows us to compute the coefficients of the equilibrium price function, backwards. Finally, starting from a steady state with homogeneous beliefs ($\mathbb{E}_{I,0}[v] = \mathbb{E}_{U,0}[v] = \mu_0$) in period $t = 0$, we simulate the model forwards.

D.3.1 Gaussian State Vector

To show that the state vector follows a Gaussian process, let's first see how each element evolves:

$$\mathbb{E}_{I,t+1}[v] = \frac{t\tau_s + \tau_0}{(t+1)\tau_s + \tau_0}\mathbb{E}_{I,t}[v] + \frac{\tau_s}{(t+1)\tau_s + \tau_0} s_{t+1} \tag{D.11}$$

$$= \mathbb{E}_{I,t}[v] + \frac{\tau_s}{(t+1)\tau_s + \tau_0}\sigma_{s_{t+1|t}} x_{t+1} \tag{D.12}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

where the second equality uses the fact that $s_{t+1} = \mathbb{E}_{I,t}[v] + \epsilon_{t+1} + (v - \mathbb{E}_{I,t}[v])$, such that $x_{t+1} \equiv \frac{\epsilon_{t+1}+(v-\mathbb{E}_{I,t}[v])}{\sigma_{t+1|t}} \sim N(0,1)$ and $\sigma_{t+1|t} \equiv (\tau_s)^{-1} + (t\tau_s + \tau_0)^{-1}$.

Turning to uninformed traders' beliefs, we assume uninformed traders form their beliefs according to (D.8) (and we also assume that informed traders are sophisticated and understand that this is how uninformed traders form beliefs):

$$\mathbb{E}_{U,t+1}[v] = \frac{1}{\tilde{A}} P_t + \frac{\tilde{K}_t}{\tilde{A}_t} \tag{D.13}$$

$$= \frac{A_t}{\tilde{A}_t}\mathbb{E}_{I,t} + \frac{B_t}{\tilde{A}_t}\mathbb{E}_{U,t}[v] - \frac{K_t - \tilde{K}_t}{\tilde{A}_t} \tag{D.14}$$

where the second equality uses our guess for the price function in (D.9).

We can now use (D.12) and (D.14) to write the evolution of the state vector as follows:

$$\Psi_{t+1} = A_{\Psi,t+1}\Psi_t + B_{\Psi,t+1}x_{t+1} \tag{D.15}$$

where $A_{\Psi,t+1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\tilde{K}_t - K_t}{\tilde{A}_t} & \frac{A_t}{\tilde{A}_t} & \frac{B_t}{\tilde{A}_t} \end{pmatrix}$ and $B_{\Psi,t+1} \equiv \begin{pmatrix} 0 \\ \frac{\tau_s}{(t+1)\tau_s+\tau_0}\sigma_{s_{t+1|t}} \\ 0 \end{pmatrix}$.

Moreover, using the definition of Q_{t+1}, and substituting in it our guessed price function in (D.9) and the law of motion of the state vector in (D.15), we have that:

$$Q_{t+1} = A_{Q,t+1}\Psi_t + B_{Q,t+1}x_{t+1} \tag{D.16}$$

where $A_{Q,t+1} \equiv A_{P,t+1}A_{\Psi,t+1} - A_{P,t}$ and $B_{Q,t+1} \equiv A_{P,t+1}B_{\Psi,t+1}$.

Since both Ψ_{t+1} and Q_{t+1} are Gaussian processes given agents' beliefs and our guessed price function, we can now apply Lemma 4 in He and Wang (1995) to show that informed traders have linear demand functions.

Electronic copy available at: https://ssrn.com/abstract=4666338

D.3.2 Informed Traders' Demand Function

Informed traders solve the following intertemporal optimization problem, according to which they maximize CARA utility over terminal wealth:

$$\max_{X_{I,t}} \quad \mathbb{E}_{I,t}\left[-e^{-\mathcal{A}W_{I,T}}\right] \quad s.t. \quad W_{I,t+1} = W_{I,t} + X_{I,t}Q_{t+1} \tag{D.17}$$

where $W_{I,T}$ is the wealth of (a single) informed trader at the final date T, and $Q_{t+1} \equiv P_{t+1} - P_t$ is the excess return on one share of the risky asset. Following He and Wang (1995), let $J(W_t;\Psi_t;t)$ be the value function. The Bellman equation for the optimization problem in (D.17) is given by:

$$0 = \max_{X_{I,t}} \{\mathbb{E}_{I,t}\left[J(W_{I,t+1};\Psi_{t+1};t+1\right] - J(W_{I,t};\Psi_t;t)\} \tag{D.18}$$

$$s.t. \quad W_{I,t+1} = W_{I,t} + X_{I,t}Q_{t+1} \tag{D.19}$$

$$J(W_{I,T};\Psi_T;T) = -e^{-\lambda W_{I,T}} \tag{D.20}$$

Since we saw in (D.15) and (D.16) that Ψ_{t+1} and Q_{t+1} are Gaussian processes, we can directly apply Lemma 4 from He and Wang (1995), given our guessed value function in (D.10). We can then show that informed traders have the following linear asset demand function:

$$X_{I,t} = \frac{1}{\mathcal{A}}F_t\Psi_t \tag{D.21}$$

where:

$$F_t \equiv (B_{Q,t+1}\Xi_{t+1}B'_{Q,t+1})^{-1}\Big(A_{Q,t+1} - B_{Q,t+1}\Xi_{t+1}B'_{\Psi,t+1}U'_{t+1}A_{\Psi,t+1}\Big) \tag{D.22}$$

$$\Xi_{t+1} \equiv \Big(1 + B'_{\Psi,t+1}U_{t+1}B_{\Psi,t+1}\Big)^{-1} \tag{D.23}$$

Plugging this demand function into the value function also allows us to verify that the

Electronic copy available at: https://ssrn.com/abstract=4666338

value function at t is of the postulated form:

$$J(W_{I,t}; \Psi_t; t) = -e^{-\mathcal{A}W_t - \frac{1}{2}\Psi_t' U_t \Psi_t} \tag{D.24}$$

with:

$$U_t = M_t + c_t I_{11}^{3,3} \tag{D.25}$$

$$M_t \equiv F_t' \left(B_{Q,t+1} \Xi_{t+1} B_{Q,t+1}' \right) F_t \\ - \left(B_{\Psi,t+1}' U_{t+1} A_{\Psi,t+1} \right)' \Xi_{t+1} \left(B_{\Psi,t+1}' U_{t+1} A_{\Psi,t+1} \right) + A_{\Psi,t+1}' U_{t+1} A_{\Psi,t+1} \tag{D.26}$$

where $c_t \equiv -2 \ln \rho_{t+1}$, $\rho_{t+1} \equiv \sqrt{|\Xi_{t+1}|}$, and $I_{11}^{3,3}$ is a (3×3) index matrix which has all the elements being zero except element $\{11\}$ being 1.[43]

Notice that (D.21) is then a function of $A_{P,t}$ (since $A_{Q,t+1}$ and $A_{\Psi,t+1}$ are both functions of $A_{P,t}$), which is the coefficient governing the price function at t. To determine these price function coefficients, we need to compute the demand function of uninformed traders, impose market clearing, and then match coefficients, given our guess in (D.9).

D.3.3 Uninformed Traders' Demand Function

The demand of uninformed traders is:

$$X_{U,t} = \frac{\mathbb{E}_{U,t}[v] - P_t}{\mathcal{A}((t-1)\tau_s + \tau_0)^{-1}} \tag{D.27}$$

[43]This adjustment is because the value function is multiplied by the constant ρ_{t+1}, independent of beliefs, which is equivalent to having the state vector multiplied by such a matrix since the first element of the state vector is just the constant 1.

Electronic copy available at: https://ssrn.com/abstract=4666338

Define the precision of uninformed agents as $\tau_{U,t} = (t-1)\tau_s + \tau_0$, so that in matrix form:

$$X_{U,t} = \frac{1}{\mathcal{A}}\tau_{U,t}\Big(\begin{pmatrix}0 & 0 & 1\end{pmatrix} - A_{P,t}\Big)\Psi_t = \frac{1}{\mathcal{A}}D_t\Psi_t \tag{D.28}$$

where $D_t \equiv \tau_{U,t}\Big(\begin{pmatrix}0 & 0 & 1\end{pmatrix} - A_{P,t}\Big)$.

D.3.4 Market Clearing and Matching Coefficients

Aggregating demands and imposing market clearing, we get:

$$Z = \left(\frac{\phi}{\mathcal{A}}F_t + \frac{1-\phi}{\mathcal{A}}D_t\right)\Psi_t \tag{D.29}$$

Since the left-hand side is a constant (Z is independent of $\mathbb{E}_{U,t}[v]$ and $\mathbb{E}_{I,t}[v]$, the second and third entries of Ψ_t), the matrix in front of Ψ_t on the right-hand side must be equal to:

$$\frac{\phi}{\mathcal{A}}F_t + \frac{1-\phi}{\mathcal{A}}D_t = \begin{pmatrix}Z & 0 & 0\end{pmatrix} \tag{D.30}$$

To isolate the unknown term $A_{P,t}$, we can decompose $A_{\Psi,t+1}$, F_t and D_t in terms that include $A_{P,t}$ and terms that do not. Specifically, let:

$$A_{1\Psi,t+1} \equiv \begin{pmatrix} \frac{t+g_t}{t+1} & 0 & \frac{(1-g_t)\mu_0}{t+1} \\ 0 & 0 & \frac{-\tilde{K}_t}{\tilde{A}_t} \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad A_{2\Psi,t+1} \equiv \begin{pmatrix} 0 \\ \frac{1}{\tilde{A}_t} \\ 0 \end{pmatrix} \tag{D.31}$$

we can then write:

$$A_{\Psi,t+1} = A_{1\Psi,t+1}A_{P,t} + A_{2\Psi,t+1} \tag{D.32}$$

$$F_t = F_{1,t}A_{P,t} + F_{2,t} \tag{D.33}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

$$D_t = D_{1,t} A_{P,t} + D_{2,t} \tag{D.34}$$

where:

$$F_{1,t} \equiv (B_{Q,t+1} \Xi_{t+1} B'_{Q,t+1})^{-1} \Big(A_{P,t+1} A_{1\Psi,t+1} - 1 - B_{Q,t+1} \Xi_{t+1} B'_{\Psi,t+1} U_{t+1} A_{1\Psi,t+1} \Big) \tag{D.35}$$

$$F_{2,t} \equiv (B_{Q,t+1} \Xi_{t+1} B'_{Q,t+1})^{-1} \Big(A_{P,t+1} A_{2\Psi,t+1} - B_{Q,t+1} \Xi_{t+1} B'_{\Psi,t+1} U_{t+1} A_{2\Psi,t+1} \Big) \tag{D.36}$$

$$D_{1,t} \equiv -\tau_{U,t} \tag{D.37}$$

$$D_{2,t} \equiv \tau_{U,t} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \tag{D.38}$$

The method of matching coefficients thus allows us to define $A_{P,t}$ recursively:

$$\frac{\phi}{\mathcal{A}} (F_{1,t} A_{P,t} + F_{2,t}) + \frac{1-\phi}{\mathcal{A}} (D_{1,t} A_{P,t} + D_{2,t}) = \begin{pmatrix} Z & 0 & 0 \end{pmatrix} \tag{D.39}$$

and since $\phi F_{1,t} + (1-\phi) D_{1,t}$ is a scalar, we can solve for the price function $A_{P,t}$ recursively as:

$$A_{P,t} = \frac{1}{\phi F_{1,t} + (1-\phi) D_{1,t}} \left(\begin{pmatrix} \mathcal{A}Z & 0 & 0 \end{pmatrix} - (\phi F_{2,t} + (1-\phi) D_{2,t}) \right) \tag{D.40}$$

D.3.5 Starting the Recursion

We need to initialize the recursion at T by providing expressions for:

1. The elements of the matrix in price function, $A_{P,T} = \begin{pmatrix} A_T & B_T & -K_T \end{pmatrix}$
2. The matrix U_T

Price Function in Period T. The price function is easy to get since there are no dynamic/speculation motives anymore in period T. Market clearing then yields:

$$\phi (T\tau_s + \tau_0) \frac{\mathbb{E}_{I,t}[v] - P_T}{\mathcal{A}} + (1-\phi)((T-1)\tau_s + \tau_0) \frac{\mathbb{E}_{U,t}[v] - P_T}{\mathcal{A}} = Z \tag{D.41}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

Which gives:

$$A_T = \frac{\phi(T\tau_s + \tau_0)}{\phi(T\tau_s + \tau_0) + (1-\phi)((T-1)\tau_s + \tau_0)} \tag{D.42}$$

$$B_T = \frac{(1-\phi)((T-1)\tau_s + \tau_0)}{\phi(T\tau_s + \tau_0) + (1-\phi)((T-1)\tau_s + \tau_0)} \tag{D.43}$$

$$K_T = \frac{\mathcal{A}Z}{\phi(T\tau_s + \tau_0) + (1-\phi)((T-1)\tau_s + \tau_0)} \tag{D.44}$$

Matrix U_T. To find U_T, notice that the expected value function of informed traders in period T is simply given by (since $U_{T+1} = 0$, as there are no more intertemporal trading motives in the final period):

$$\mathbb{E}_{I,T} \exp\left(-\mathcal{A}W_{T+1}\right) = \mathbb{E}_{I,t} \exp\left(-\mathcal{A}[W_T + X_{I,T}(v - P_T)]\right) \tag{D.45}$$

where only v is stochastic, and follows a normal distribution: $v \sim \mathcal{N}\left(\mathbb{E}_{I,T}[v], \frac{1}{T\tau_s + \tau_0}\right)$. So this is simply equal to:

$$\mathbb{E}_{I,T} \exp\left(-\mathcal{A}W_{T+1}\right) = \exp\left(-\mathcal{A}[W_T - X_{I,T}P_T]\right) \mathbb{E}_T \exp\left(-\mathcal{A}X_{I,T}v\right) \tag{D.46}$$

$$= \exp\left(-\mathcal{A}[W_T - X_{I,T}P_T]\right) \exp\left(-\mathcal{A}X_{I,T}\mathbb{E}_{I,T}[v] + \frac{1}{2}\frac{\mathcal{A}^2 X_{I,T}^2}{T\tau_s + \tau_0}\right) \tag{D.47}$$

For conciseness let $I_{EI} \equiv \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$, so that the demand function can be written as:

$$X_{I,T} = \frac{1}{\mathcal{A}} F_T \Psi_T \text{ where } F_T = \left(T\tau_s + \tau_0\right)\left(I_{EI} - A_{P,T}\right) \tag{D.48}$$

and the various components in (D.47) can be written as:

$$\mathcal{A}X_{I,T}P_T = F_T \Psi_T A_{P,T} \Psi_T = \frac{1}{\mathcal{A}} \Psi_T' F_T' A_{P,T} \Psi_T \tag{D.49}$$

Electronic copy available at: https://ssrn.com/abstract=4666338

$$\mathcal{A}X_{I,T}\mathbb{E}_{I,T}[v] = \Psi_T' F_T' I_{EI}\Psi_T \tag{D.50}$$

$$\frac{1}{2}\frac{\mathcal{A}^2 X_{I,T}^2}{T\tau_s + \tau_0} = \frac{1}{2(T\tau_s + \tau_0)}\Psi_T' F_T' F_T \Psi_T \tag{D.51}$$

Substitututing (D.49), (D.50) and (D.51) into (D.47), we have:

$$\mathbb{E}_T \exp\left(-\mathcal{A}W_T - \frac{1}{2}\Psi_T' U_T \Psi_T\right) \tag{D.52}$$

where the first iteration of the U matrix is pinned down as follows:

$$U_T = -2F_T'\left(A_{P,T} - E_{T,I} + \frac{1}{2(T\tau_s + \tau_0)}F_T\right) \tag{D.53}$$

which concludes the recursion.

D.3.6 Numerical Solution

To simulate a price path for the intertemporal problem, we proceed as follows:

1. For each t, construct the misspecified model of the world used by Uninformed traders according to equation (D.7), in order to recover $\tilde{A}_t$, $\tilde{K}_t$;

2. Construct the matrices $A_{P,T}$ and U_T that initiate the recursion according to equations (D.42), (D.43), (D.44) and (D.53);

3. Recursively construct $A_{P,t}$ and U_t for each t by backward induction;

4. Starting from a steady state with $\mathbb{E}_{I,0}[v] = \mathbb{E}_{U,0}[v]$, feed a path for signals $\{s_t\}$ and construct the price path forwards.

Electronic copy available at: https://ssrn.com/abstract=4666338

D.4 Results

The top panel of Figure 6 compares the equilibrium price path with intertemporal trading motives achieved in this way (green line) to the price path which arises when all traders have period-by-period mean-variance utility and uninformed traders are either rational (red line) or partial equilibrium thinkers (blue line). The left panel depicts equilibrium prices in normal times, as shown from the fact that the corresponding strength of the feedback effect (depicted in the bottom left panel) is always below one. The right panel depicts equilibrium prices following a displacement, as shown from the fact that the corresponding strength of the feedback effect (depicted in the bottom right panel) temporarily increases above one.[44]

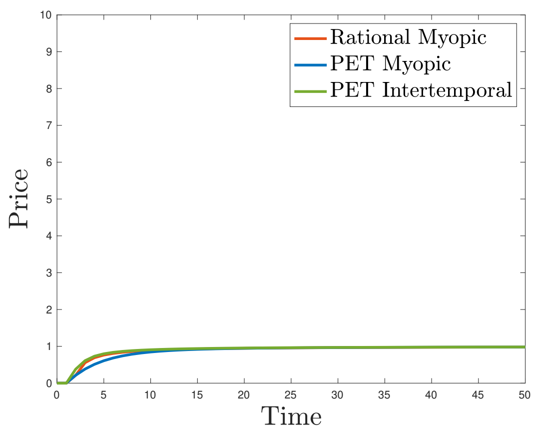
Figure 6 shows that in normal times dynamic trading motives lead informed traders to arbitrage the short-term mispricing away more quickly, than when traders are myopic. Instead, following a displacement, dynamic trading motives amplify the bubble. These results are consistent with the intuition we uncovered in Section 3, where traders had speculative motives. To understand why this is the case, notice that when informed traders have dynamic trading motives, they understand that a higher price today leads uninformed traders to have more optimistic beliefs tomorrow, thus pushing up potential capital gains from holding the asset today. This leads to a higher $\Psi_t U_t \Psi_t$ in the value function. As a result, informed trader's marginal utility of present wealth is lower, which makes it attractive for them to buy the asset (and which effectively makes their asset demand more inelastic), pushing up the price further, and making speculation even more attractive. This generate a two-way feedback effect between prices and expected capital gains, which amplifies the two-way feedback effect between prices and uninformed traders' beliefs inherent in partial equilibrium thinking.

[44] As in the baseline model, the strength of the feedback effect is stronger when there are fewer informed traders in the market, and when the informativeness of news is low relative to the prior.

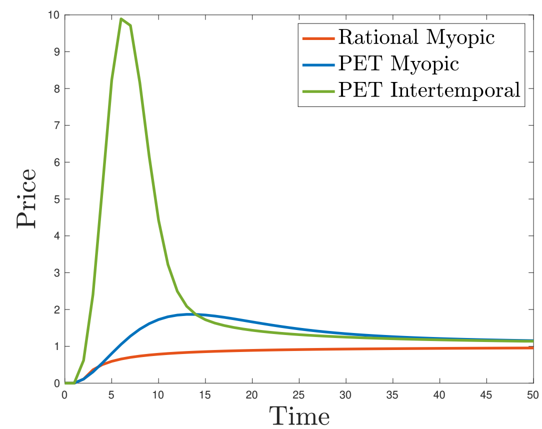
Electronic copy available at: https://ssrn.com/abstract=4666338

Figure 6: Bubbles and crashes with intertemporal trading. The left panel simulates price paths and the corresponding feedback effect after normal times shock, when the feedback effect stays below 1 throughout. The right panel simulates the price path and the corresponding feedback effect after a displacement shock, when the strength of the feedback effect temporarily increases above 1. The green lines plot equilibrium prices when informed traders have intertemporal trading motives, while the blue and red lines plot equilibrium price paths when all traders have period-by-period mean variance utility, under partial equilibrium thinking and under rational expectations respectively.

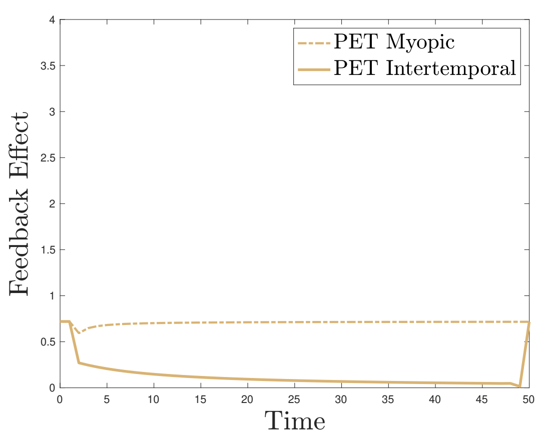
(a) Price after normal time shocks



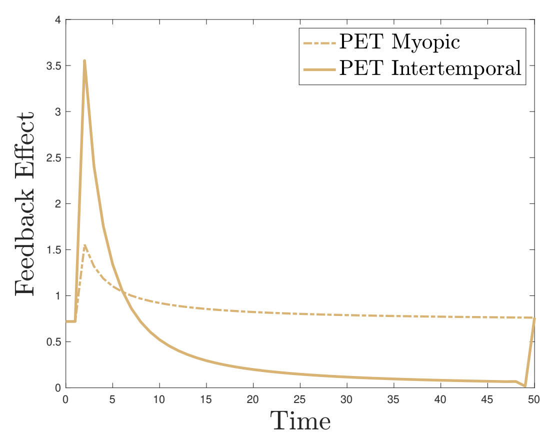
(b) Price after displacement



(c) Feedback effect after normal time shocks



(d) Feedback effect after displacement



Electronic copy available at: https://ssrn.com/abstract=4666338