Time-Averaging and Taxation[*]

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1 Introduction

This paper extends a twenty-year old aggregate labor supply paradigm shift from the Rogerson (1988) "employment lotteries" that Prescott (2005) said was an essential ingredient of his 2004 Nobel lecture to the "time averaging" theory that Prescott instead embraced when he updated his Nobel lecture (Prescott 2006*b*).[1] After describing the microeconomic evidence and economic logic that galvanized the paradigm shift, this paper uses a time-averaging theory to evaluate reforms that implement either a flat-rate tax, a negative income tax, or a universal basic income. After presenting a standard *ex ante* welfare analysis, we explore an *ex post*-*ex ante* welfare measure that expresses concerns about individuals who *ex post* find themselves in especially disadvantageous states, taking into account the diverse lifetime experiences that put people in those states.

To appreciate the microeconomics that influenced the paradigm shift, recall how Prescott (2005, p. 385) used an aggregation theory of Rogerson (1988) that he said "is every bit as important as the one giving rise to the aggregate production function" to infer a high labor supply elasticity from aggregate employment fluctuations observed over business cycles. Rogerson obtained a high aggregate labor supply elasticity by combining (i) indivisible labor, with (ii) employment lotteries together with complete markets for insuring individual agents' consumption against the risk injected by those employment lotteries. When equilibrium employment-to-population ratios are less than one, those lotteries yield a large aggregate labor supply elasticity by convexifying individuals' labor supply choices. But micro-labor critics of Rogerson's aggregation theory saw no direct micro evidence for components (ii). Thus, Browning, Hansen & Heckman (1999, p. 602) wrote that the "employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories."

Motivated partly by those criticisms, Ljungqvist & Sargent (2006) proposed and ana-

[1] See the added section "The Life Cycle and Labor Indivisibility" in Prescott (2006*b*). To learn more about Prescott's attitude about the significance of this paradigm shift, see his discussion (Prescott 2006*a*) of Ljungqvist & Sargent (2006).

lyzed the time-averaging theory that was adopted by their discussant Prescott (2006*a*). Like the employment-lottery theory, it delivers a high aggregate labor supply elasticity although it discards Rogerson's employment lotteries and his assumption of complete consumption-insurance markets. The time-averaging theory replaces those components with a life cycle model of labor supplies and intertemporal trades in non-state-contingent credit markets. Thus, in a continuous-time, non-stochastic life-cycle incomplete-market economy that retains Rogerson's indivisible labor component (i), Ljungqvist and Sargent deduced the same individual (expected) utilities, aggregate allocation and high aggregate labor supply elasticity that prevail in a Rogerson complete-market economy with employment lotteries.[2] In place of a representative *family* that chooses probabilities that individual family members work at each point in time, an individual is on his own and chooses a fraction of a lifetime to devote to work and how much to save and spend. A worker uses a credit market to smooth consumption across episodes of work and times of not working, perhaps called retirement. Among other things, Ljungqvist & Sargent (2006) showed that time averaging can replace lotteries and still deliver a high aggregate labor supply elasticity.[3] Their discussant Prescott (2006*a*, p. 233) praised "the initiation of an important research program ... to derive the implications of labor indivisibility for lifetime labor supply ... a program that already has begun to bear fruit."[4]

Shifting to the time-averaging paradigm makes each individual worker responsible for allocating his time between work and leisure each period in light of his financial wealth and

[2]In terms of technical conditions, under time averaging, a high labor supply elasticity no longer merely requires an employment-to-population ratio less than one; instead it requires workers to be at interior solutions for their choices of career lengths.

[3]Independently, Chang & Kim (2006) discovered a high aggregate labor supply elasticity in simulations of a stochastic Bewley model with incomplete markets and indivisible labor. Their agents optimally alternate between periods of work and leisure (they "time average") to allocate consumption and leisure over their infinite lifespans.

[4]Prescott, Rogerson & Wallenius (2009) extended the Ljungqvist & Sargent (2006) framework by adding an intensive margin to the individual's labor supply decision. Given a constant wage over the life cycle, Prescott et al. affirmed Ljungqvist & Sargent's outcome that all adjustments in labor supply in response to taxation take place at the extensive margin. While Rogerson & Wallenius (2009) proceeded to activate the intensive margin by assuming exogenous life cycle variation in the wage – independent of a worker's labor market experience, the overarching conclusion remained that the extensive margin of labor supply is most critical for the magnitude of the aggregate labor supply elasticity.

opportunities to borrow and lend. Relative to the employment-lottery model, it facilitates bringing to bear new evidence, including observations that concerned Browning et al. (1999). For example, Ljungqvist & Sargent (2014) studied how career lengths depend on shapes of the earnings profile, earnings shocks, taxes, and aspects of government financed retirement schemes.[5] Ljungqvist & Sargent (2008) investigated general equilibrium effects of taxation and government supplied non-employment benefits in a life cycle model with human capital accumulation under both employment lotteries and time averaging, respectively. While many aggregate outcomes are similar in the two paradigms, there can be stark microeconomics differences in who works and who doesn't. These include dubious outcomes under employment lotteries that have workers with successful accumulation of human capital being destined never to retire.[6] When he discussed Ljungqvist & Sargent (2008), Rios-Rull (2008) advocated incomplete-market, heterogeneous-agent models and recommended abandoning representative-agent models, including those founded on the employment-lottery paradigm. Rios-Rull conjectured that advantages brought by the time-averaging model be the "reason Rogerson (Rogerson & Wallenius, 2007 [working paper for 2009 article]) is now using OLG models without lotteries to address the employment question."

When Prescott had still relied on employment lotteries as an important pillar of his aggregate model, the magnitude of the aggregate labor supply elasticity separated Prescott (2002) (large) from Heckman (1993) (very small). The shift of macro-labor economists to the time-averaging life-cycle models fosters a potential reconciliation about different magnitudes of

[5]Under the auxiliary assumption that preferences are consistent with balanced growth, Ljungqvist & Sargent (2014) obtain stark outcomes under time averaging. For example, off corners, the more elastic are earnings to accumulated working time, the longer is a worker's career. This result suggests the possibility that it is a higher *slope* of the earnings-experience profile of high wage workers, and not the *level* of the wage *per se*, that explains why people with higher wages and higher educations are more likely to retire later in life. Evidence for such a relationship is provided by a study of married women's labor force participation by Eckstein & Wolpin (1989).

[6]Such incredible features of employment-lottery allocations were noted earlier, for example, when Ljungqvist & Sargent (2006, fn. 22) thanked "Richard Rogerson for alerting us to Grilli & Rogerson (1988) who also analyze human capital accumulation in a model with employment lotteries. The authors cite the story 'The Lottery in Babylon' by the surrealist Jorge Luis Borges, in which an all-encompassing lottery dictates all activities in a fictional society. The Borges story either arouses skepticism about the real-world relevance of the analysis or exemplifies that reality sometimes surpasses fiction."

the aggregate labor supply elasticity inferred by Prescott (large) and Heckman (very small). Actually, even back when Prescott and other real business cycle theorists had still relied on employment lotteries, a distinct life-cycle approach to modelling aggregate labor supply was also widespread. However, that work often imposed an exogenous retirement age. Examples occurred in macroeconomic applications of life-cycle models dating back at least to Auerbach & Kotlikoff's (1987) classic extension of the overlapping-generations structure of Diamond (1965) and Samuelson (1958) for quantitative policy analysis.[7] Heckman, Lochner & Taber (1998) also assumed an exogenous retirement age in their macro-labor analysis of schooling choices and Ben-Porath human capital accumulation in a life cycle model. But hard-wiring retirement ages disarms a force that can help generate a high aggregate labor supply elasticity. To unleash that force, career lengths and retirement ages have to be endogenous. Allowing them to be objects that individual workers choose opens the door for realizing the vision of Browning, Hansen & Heckman (1999, p. 625): "Macroeconomic theory will be enriched by learning from . . . empirical research in microeconomics [and] microeconomics will be enriched by . . . dynamic general equilibrium theory."

To foster such a reconciliation, this paper starts from the "Prescott side" by modifying a structure of by Holter, Krueger & Stepanchuk (2019, henceforth HKS) that had only partially implemented time-averaging. We modify that model to activate time-averaging fully, then use the model to analyze tax reforms.[8] Dirk Krueger, the coauthor of HKS, is someone

[7] It is noteworthy that the coexistence of the employment-lottery paradigm and the life cycle approach at the time did not generate much of a discussion about their starkly different implications for the aggregate labor supply elasticity. Ljungqvist & Sargent (2008, fn. 21) even suggested that "[t]he current state of affairs in macroeconomics between the representative-agent framework and heterogeneous-agent models is best described as an 'harmonious' one. For example, Prescott (2006*b*) cites that the importance of total factor productivity shocks for business cycle fluctuations, as estimated in his representative agent model, is robust in the alternative heterogenous-agent models of Ríos-Rull (1994) and Krusell & Smith (1998)" where the overlapping generations of Ríos-Rull are subject to an exogenous retirement age and the infinitely-lived agents of Krusell & Smith transition exogenously between work and nonwork.

[8] In the same spirit, Graves, Gregory, Ljungqvist & Sargent (2023) start from the "Heckman side" by unleashing time-averaging forces that were only latent in Heckman, Lochner & Taber (1998). A key finding is that the mechanism of Ljungqvist & Sargent (2006) in our footnote 5 is evidently lurking in the Heckman et al. framework with a Ben-Porath technology for human capital accumulation. Since the technology for college workers is estimated to be more productive, college workers are prone to choose longer careers than high school workers in a time-averaging version of Heckman et al. (1998).

who was sympathetic to the “Prescott side” of using employment lotteries.[9] HKS’s general equilibrium model with heterogeneous agents and marital dynamics is a good laboratory for studying the welfare effects of tax reform. We model transitions between singlehood and marriage as exogenous shocks, as suggested by Cubeddu & Rios-Rull (2003), who also assume that both marriage partners abide by decision rules chosen by the couple. In addition, HKS assume that the flow utility of each spouse changes so that the pleasure of consumption and joint hardship of work are felt equally by both parties, with primitives specified in ways that lend financial advantages to being married for both spouses, except after a divorce when it is possible that our assumption that the couple’s assets are split equally can be disadvantageous to one of the soon-to-be-former spouses.

Section 2 sets out our model, while Section 3 describes calibration and estimation. Section 4 conveys mechanisms and forces at work. Tax reforms are analyzed in Section 5. Our analysis of different tax systems takes advantage of Prescott’s (2002) insight that the effects of taxation depend sensitively on whether the government spends tax revenues in ways that are close substitutes to private consumption or on public goods that are not. Nevertheless, our application brings out important differences that arise from Prescott’s using an employment-lottery theory, on the one hand, and our using a time-averaging theory, on the other hand. Section 6 offers concluding remarks.

2 The Model

Our model is a full-fledged time-averaging extension of the life-cycle, heterogeneous-agent framework with 1- and 2-person households in HKS. In the HKS model, both genders have intensive labor supply margins, but only females have an extensive margin. The HKS model also imposes exogenous retirement at age 65 for both genders. We drop HKS’s assumption of exogenous retirement at age 65 and include extensive margins of labor supply for both

[9]See Krueger (2007, sec. 9.2) for an enlighening exposition of the Rogerson (1988) employment-lottery aggregation theory.

men and women. Both genders face endogenous wage profiles that depend on their years of labor market experience. To attenuate labor supply responses in old age, we follow Graves, Gregory, Ljungqvist & Sargent (2023) and assume that efficiency units of experience depreciate with age, calibrated to be especially noticable fir workers in their late 60s. As for government policy, we drop HKS's assumption of a fixed unemployment benefit to women and replace HKS's fixed social security benefit to retirees with benefits that depend on labor earnings and years worked. We retain most other components of HKS.[10]

2.1 Technology

A representative firm operates a Cobb-Douglas production function

$$Y_t(K_t, L_t) = K_t^{\alpha} \left[Z_t L_t\right]^{1-\alpha},$$

where K_t is capital, L_t is the labor measured in efficiency units, and Z_t is labor-augmenting productivity. Physical capital evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t,$$

where I_t is gross investment, and δ is the capital depreciation rate. Productivity Z_t grows deterministically at rate $\mu : Z_t = (1+\mu)^t$, starting from $Z_0 = 1$. In each period, a representative firm rents labor and capital in amounts that maximize profits:

$$\Pi_t = Y_t - w_t L_t - (r_t + \delta)K_t.$$

[10] We have amended the HKS formulation of households' optimization problems. HKS correctly formulated the problem of a married couple in the form of a unitary household that by assumption places equal weights on spouses' continuation values in singlehood after a divorce. However, working backward, HKS used the value function of a unitary household to represent the value of the two individuals entering into that marriage. That formulation effectively meant that the spouses were randomizing over the genders they would take after a divorce. We instead deduce an authentic spouse-specific value function from the policy function of a married household together with the continuation value at a divorce by the appropriate spouse's value function in singlehood. See Appendix A.3 for further discussion and comparisons to HKS.

In competitive equilibrium, factor prices equal marginal products:

$$w_t = \partial Y(K_t, L_t)/\partial L_t = (1-\alpha)Z_t^{1-\alpha}\left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha)Z_t\left(\frac{K_t/Z_t}{L_t}\right)^{\alpha} \tag{1}$$

$$r_t = \partial Y(K_t, L_t)/\partial K_t - \delta = \alpha Z_t^{1-\alpha}\left(\frac{L_t}{K_t}\right)^{1-\alpha} - \delta = \alpha\left(\frac{L_t}{K_t/Z_t}\right)^{1-\alpha} - \delta \tag{2}$$

We restrict attention to balanced growth equilibria. Exogenous technological progress generates persistent growth. Following King et al. (2002) and Trabandt & Uhlig (2011), we impose restrictions on the production technology, preferences, and government policies that, after standard transformations, allow us to work with stationary variables. Along a balanced growth path (BGP) $K^z = K_t/Z_t$ is constant. We define $w_t^z = w_t/Z_t$ and note that w_t^z and r_t are constant along a BGP. Therefore, we drop time subscripts for these variables as well.

2.2 Demographics

There are J overlapping generations of finitely lived households, with household age indexed by $j \in J$. Data show that family type is an important determinant of labor supply elasticities, so we model heterogeneity in family structure explicitly.[11] Households are either single (denoted by S) or married (denoted by M); single households differ by gender (man or woman), denoted $\iota \in (m, w)$. Thus, there are 3 types of households: single males, single females, and married couples. We assume that within a married household, husband and wife are the same age. All households start life at age 20 and work until at least age 65, the first age at which social security can be collected.

A model period is one year. The probability of dying before age 65 is zero. Households aged 65 and older face age-dependent probabilities of dying, $\pi(j)$ until age $J = 81$, when they die for sure. Age $J = 81$ in our model translates to a biological age of 100. Husband and wife die at the same time. We assume that the size of the population is fixed and normalize the size of each newborn cohort to 1. Using $\omega(j) = 1 - \pi(j)$ to denote an age-dependent survival probability, application of a law of large numbers implies that the mass of agents

[11] Keane (2011) stresses the importance of marital status in shaping labor supply responses to taxes.

of age $j \geq 65$ still alive is $\Omega_j = \prod_{q=65}^{q=j-1} \omega(q)$. A fraction of households leave unintended bequests that are allocated to surviving agents in the same cohort. It is as if individuals in each cohort own shares of a mutual fund, with shares of dying owners being distributed proportionally among surviving owners. Surviving retirees of age j then earn gross return $(1+r)/\omega(j-1)$ on savings.

In addition to age and marital status, households are heterogeneous with respect to asset holdings k, exogenously determined permanent abilities $a \sim N(0, \sigma_a^{\iota 2})$ drawn at birth, years of labor market experience e, and idiosyncratic productivity shocks u. There are extensive and intensive abor supply margins. Individuals choose either to work or to stay at home; conditional on working, they choose how much to work. Married households jointly decide on how many hours to work, how much to consume, and how much to save. Individuals who participate in the labor market accumulate a year of labor market experience. Individuals choose when to retire. The earliest possible retirement age is 65. Retired individuals receive social security benefits and don't work.

Since labor supply decisions differ across family type and ages, we want to use an empirically plausible joint distribution of family types and ages. A tractable approach is to introduce marriage and divorce as exogenous shocks, as in Cubeddu & Rios-Rull (2003) and Chakraborty et al. (2015). Single households face an age-dependent probability $M(j)$ of becoming married and married households face an age-dependent probability $D(j)$ of divorce. Assortative matching in the marriage market means that there is a greater chance of marrying someone with similar ability. Thus, a single male with ability a^m faces a probability $\phi^w(a|a^m;\varphi)$ of marrying a female of type a, and symmetrically, a female of type a^w marries a male of ability a with probability $\phi^m(a|a^w;\varphi)$. Parameter φ, calibrated in section 3, captures the degree of sorting in the marriage market, with $\varphi = 0$ standing in for perfectly random marriage and $\varphi = 1$ representing perfect sorting by permanent ability.[12]

[12]Conditional on gender, age and permanent ability, a single household expects to draw a partner drawn from the joint distribution of age-specific characteristics of single people of the other gender. Thus, since permanent ability and assets are positively correlated for single females, a single male understands that if he were, by chance, to marry a high ability female, she would bring higher than average assets into the

2.3 Wages

An individual's wage depends on the aggregate wage per efficiency unit of labor, $w^z = \frac{w}{Z}$, and his or her endowment of efficiency units, which depends on the individual's gender, $\iota \in (m, w)$, ability, a, accumulated labor market experience, e, age, and an idiosyncratic shock u that follows an AR(1) process. Thus, the wage of an individual with characteristics (a, e, u, ι) is

$$log(\widetilde{w}^z(a, e, u, \iota)) = log(w^z) + a + \gamma_0^\iota + \gamma_1^\iota e + \gamma_2^\iota e^2 + \gamma_3^\iota e^3 + u \tag{3}$$

$$u' = \rho^\iota u + \epsilon, \quad \epsilon \sim N(0, \sigma^2_{\epsilon^\iota}) \tag{4}$$

Parameters γ_0^ι encode average productivity, while γ_1^ι, γ_2^ι and γ_3^ι describe returns to experience for women and men. To describe depreciation of human capital in old age, let

$$\varepsilon(j) = \frac{1}{1 + \exp(\phi_1(j - \phi_2))} \leq 1$$

be a multiplicative factor that transforms the human capital stock of an agent of age j into efficiency units.

An individual of age j's wage adjusted for human capital depreciation is:

$$w^z(a, e, u, \iota, j) = \widetilde{w}^z(a, e, u, \iota) \times \varepsilon(j)$$

2.4 Preferences

Married couples solve a joint maximization problem with equal weights on spouses' one-period utilities. The one-period utility function depends on joint consumption c, hours

marriage.

$n^m \in [0,1]$ worked by the husband and hours $n^w \in [0,1]$ worked by the wife and has form:

$$U^M(c, n^m, n^w) = \log(c) - \frac{1}{2}\chi_M^m \frac{(n^m)^{1+\eta^m}}{1+\eta^m} - \frac{1}{2}\chi_M^w \frac{(n^w)^{1+\eta^w}}{1+\eta^w} - \frac{1}{2}F_M^m \cdot \mathbb{1}_{[n^m>0]} - \frac{1}{2}F_M^w \cdot \mathbb{1}_{[n^w>0]} + \log(G). \tag{5}$$

Here $\log(F_M^\iota)$ is a fixed disutility from working positive hours and G is the public good . The indicator function $\mathbb{1}_{[n>0]}$ equals 0 when $n = 0$ and 1 when $n > 0$. The period utility function for singles is:

$$U^S(c, n, \iota) = \log(c) - \chi_S^\iota \frac{(n)^{1+\eta^\iota}}{1+\eta^\iota} - F_S^\iota \cdot \mathbb{1}_{[n>0]} \tag{6}$$

Disutility of work and the fixed cost of work can differ by gender and marital status. In a model without participation margin, King et al. (2002) show that the above preferences are consistent with balanced growth. HKS demonstrate that this is also true in a model with a fixed utility cost from working positive hours and an operative extensive margin.

2.5 Government

The government runs a balanced budget period-by-period. It taxes workers and the representative firm at rates τ_{ss} and $\tilde{\tau}_{ss}$, consumption at flat rate τ_c, capital income at flat rate τ_k, and labor at a progressive tax rate that we shall soon describe. The government spends for pure public consumption goods G_t, interest payments rB_t on the national debt, lump sum redistributions g_t, and benefits Ψ_t to retirees. We assume that there is some outstanding government debt, and that the government debt to output ratio, $B_Y = B_t/Y_t$, is constant over time. We assume that spending on public consumption is also proportional to GDP, so $G_Y \equiv G_t/Y_t$ is constant. In the U.S., interest income is taxed together with labor income and the corporate tax code is non-linear. Nevertheless, we follow a common practice that approximates a capital income tax schedule with a flat tax.

We allow social security benefits to depend on an individual's gender, marital status, $q \in \{M, S\}$, innate ability (level of education), and years of labor market experience. After reaching the official retirement age of 65, individuals decide whether to retire. If they choose to retire, they can begin to collect social security pension payments that satisfy the formula

$$\Psi(\iota, q, a_i, e_i) = \psi_0 + \psi_1 \times AE(\iota, q, a_i) \times \min\{1, e_i/35\}, \tag{7}$$

where $AE(\iota, m, a_i)$ is average earnings of individuals with the indicated gender, marital status and ability, where the average is over ages 30-64. The pension payment is increasing in experience up to 35 years of experience; ψ_0 is a minimum pension benefit, while ψ_1 determines how benefits increase with lifetime earnings.

If an individual decides to continue working, he or she cannot collect the pension payments. The implied implicit tax on the labor supply of individuals who have reached their retirement age can put some of them the corner solution that tells them to retire at age 65. In a tax reform, we will study effects on the aggregate labor supply elasticity from removing that implicit tax.

To model the non-linear labor income tax discussed in Section 2, we use a function of Benabou (2002) that was also used by Heathcote et al. (2017). This function makes the average tax rate on labor income y be $\tau(y) = 1 - \theta_0 y^{-\theta_1}$; parameters θ_0 and θ_1 govern the level and the progressivity of the tax system.

We use superscript Z to denote aggregate variables deflated by total factor productivity Z. Thus, we define deflated tax revenue from social security, labor, capital and consumption taxes, R^z, transfers, g^z, government consumption, G^z, and social security benefits, Ψ^z, as:

$$R^z = R_t/Z_t, \quad g^z = g_t/Z_t, \quad G^z = G_t/Z_t, \quad \Psi^z = \Psi_t/Z_t$$

Along a BGP these variables are constant shares of GDP. The government budget constraint (normalized by the level of technology) along a BGP is:

$$\Psi^z + g^z \left(45 + \sum_{j \geq 65} \Omega_j\right) + G^z + (r - \mu)B^z = R^z$$

The government spends resources on social security benefits, transfers, government consumption, and servicing outstanding government debt. It uses tax revenue to pay for that.

2.6 Recursive Formulation of Household Problem

A pre-retirement-age married household is characterized by the household's age j, its assets, k, the man's and the woman's experience levels, e^m, e^w, their transitory productivity shocks u^m, u^w and permanent ability levels a^m, a^w. Thus, the state vector for such a married household is $(k, e^m, e^w, u^m, u^w, a^m, a^w, j)$. The state vector for a single household is (k, e, u, a, ι, j). To formulate a household's problem along a BGP recursively, we define deflated household consumption and assets as $c^z = c_t/Z_t$ and $k^z = k_t/Z_t$. Since ratios of aggregate variables to productivity, Z_t, and to aggregate output are constant along the BGP, we posit that household-level variables, c^z and k^z, do not depend on calendar time, we omit the time subscripts for them.

For married couples, we keep track of two value functions. We follow Cubeddu & Rios-Rull (2003) and assume that during marriage, both partners obey decision rules chosen by a unitary household, in particular, ones that attain the optimal value function that satisfies the following Bellman equation:

$$
\begin{aligned}
&V^M(k^z, e^m, e^w, u^m, u^w, a^m, a^w, j) = \max_{c^z,(k^z)',n^m,n^w} \Big[U^M(c^z, n^m, n^w) \\
&+ \beta(1 - D(j)) E_{(u^m)',(u^w)'} \left[V^M((k^z)', (e^m)', (e^w)', (u^m)', (u^w)', a^m, a^w, j+1) \right] \\
&+ \frac{1}{2} \beta D(j) E_{(u^m)',(u^w)'} \left[V^S((k^z)'/2, (e^m)', (u^m)', a^m, m, j+1) + V^S((k^z)'/2, (e^w)', (u^w)', a^w, w, j+1) \right] \Big]
\end{aligned} \tag{8}
$$

where maximization is subject to

$$
\begin{aligned}
&c^z(1+\tau_c) + (k^z)'(1+\mu) = k^z\left(1 + r(1-\tau_k)\right) + 2g^z + Y^L \\
&Y^L = \left(Y^{L,m} + Y^{L,w}\right)\left(1 - \tau_{ss} - \tau_l^M\left(Y^{L,m} + Y^{L,w}\right)\right) \\
&Y^{L,\iota} = \frac{n^\iota w^{z,\iota}\left(a^\iota, e^\iota, u^\iota\right)}{1 + \tilde{\tau}_{ss}}, \ \iota = m, w \\
&(e^m)' = e^m + \mathbb{1}_{[n^m>0]}, \quad (e^w)' = e^w + \mathbb{1}_{[n^w>0]}, \\
&n^m \in [0,1], \quad n^w \in [0,1], \quad (k^z)' \geq 0, \quad c^z > 0
\end{aligned}
$$

Y^L is household labor income, composed of labor incomes that spouses receive during the working phase of their life, τ_{ss} and $\tilde{\tau}_{ss}$ are social security contributions paid by employee and employer.

In addition to value function (8), we want value functions of partners within couples. We require these objects to commpute the values of getting married for single individuals. To define them, let $\widehat{c}^z(.), (\widehat{k}^z(.))', \widehat{n}^m(.), \widehat{n}^w(.)$ be the optimal policy functions for consumption, savings and hours worked that attain the value function (8) for married households at a given state vector $(\cdot)$. Value functions of partners within couples are:

$$
\begin{aligned}
&\widetilde{V}^M(k^z, e^m, e^w, u^m, u^w, a^m, a^w, \iota, j) = \Big[U^M\left(\widehat{c}^z(.), \widehat{n}^m(.), \widehat{n}^w(.)\right) \\
&+ \beta(1 - D(j)) E_{(u^m)',(u^w)'} \left[\widetilde{V}^M((\widehat{k}^z(.))', e^m + \mathbb{1}_{[\widehat{n}^m(.)>0]}, e^w + \mathbb{1}_{[\widehat{n}^w(.)>0]}, (u^m)', (u^w)', a^m, a^w, \iota, j+1) \right] \\
&+ \beta D(j) E_{(u^\iota)'} V^S((\widehat{k}(.)^z)'/2, e^\iota + \mathbb{1}_{[\widehat{n}^\iota(.)>0]}, (u^\iota)', a^\iota, \iota, j+1) \Big]
\end{aligned} \tag{9}
$$

where V^S is the value function of a single household to be defined below in equation (10). No optimisation appears on the right side of equation (9): we simply plug in optimal decisions that attain value function (8) for couples.

A single person of gender ι knows probabilities of marrying someone of opposite gender $-\iota$. That person's value function satisfies the Bellman equation:

$$\begin{aligned} V^S(k^z, e, u, a, \iota, j) = \max_{c^z,(k^z)',n} \Big[& U^S\left(c^z, n, \iota\right) \\ & + \beta(1 - M(j))E_{u'}\left[V^S((k^z)', e', u', a, \iota, j+1)\right] \\ & + \beta M(j) E_{(k^{-\iota})', e^{-\iota}, (u^m)', (u^w)', a^{-\iota}}\left[\widetilde{V}^M((k^z)' + (k^{-\iota})', (e^w)', (u^m)', (u^w)', a^m, \end{aligned} \tag{10}$$

where maximization is subject to

$$\begin{aligned} & c^z(1+\tau_c) + (k^z)'(1+\mu) = k^z\left(1 + r(1-\tau_k)\right) + g^z + Y^L \\ & Y^L = \left(Y^{L,\iota}\right)\left(1 - \tau_{ss} - \tau_l^S\left(Y^{L,\iota}\right)\right) \\ & Y^{L,\iota} = \frac{n^\iota w^{z,\iota}\left(a^\iota, e^\iota, u^\iota\right)}{1 + \tilde{\tau}_{ss}}, \ \iota = m, w \\ & (e^\iota)' = e^\iota + \mathbb{1}_{[n^\iota > 0]}, \\ & n^\iota \in [0,1], \quad (k^z)' \geq 0, \quad c^z > 0 \end{aligned}$$

$E_{(k^{-\iota})', e^{-\iota}, (u^m)', (u^w)', a^{-\iota}}$ is a conditional expectation over the joint distribution of characteristics of a partner in the case of marriage as well as this individual's labor productivity next period. The joint distribution is conditional on the individual's age and permanent ability.[13]

When an individual reaches age 65, two important things occur: (1) the individual chooses whether to retire; (2) there will be neither marriage or divorce: if the person is married, it really is "not until death do we part" (this is a pretty good approximation to empirical outcomes). An individual who retires starts collecting pension payments. Individuals of ages 65 and older ($j \geq 65$) carry a state variable $\Lambda \in \{0, 1\}$ that indicates retirement status.

[13]There is perfect assortative matching with respect to age, and, to some (calibrated) extent, with respect to permanent ability.

Retirement is irreversible, meaning that somene cannot retire, receive benefits, then return to work. The value function of married couples aged 65 and older satisfies the Bellman equation:

$$
\begin{aligned}
&V^M(k^z, e^m, e^w, u^m, u^w, a^m, a^w, \Lambda^m, \Lambda^w, j) = \max_{c^z,(k^z)',n^m,n^w,(\Lambda^m)',(\Lambda^w)'} \Big[U^M\left(c^z, n^m, n^w\right) \\
&+ \beta E_{(u^m)',(u^w)'}\big[V^M((k^z)', (e^m)', (e^w)', (u^m)', (u^w)', a^m, a^w, (\Lambda^m)', (\Lambda^w)', j+1)\big] \qquad (11)
\end{aligned}
$$

where maximization is subject to

$$c^z(1+\tau_c) + (k^z)'(1+\mu) = k^z\left(1 + r(1-\tau_k)\right) + 2g^z + Y^L + \Psi^{zm}\Lambda^m + \Psi^{zf}\Lambda^f$$

$$Y^L = \left(Y^{L,m} + Y^{L,w}\right)\left(1 - \tau_{ss} - \tau_l^M\left(Y^{L,m} + Y^{L,w}\right)\right)$$

$$Y^{L,\iota} = \frac{n^\iota w^{z,\iota}\left(a^\iota, e^\iota, u^\iota\right)}{1+\tilde{\tau}_{ss}}, \ \iota = m, w$$

$$(e^m)' = e^m + \mathbb{1}_{[n^m>0]}, \quad (e^w)' = e^w + \mathbb{1}_{[n^w>0]},$$

$$\text{If } \Lambda^\iota = 0, \quad n^\iota \in [0,1], \text{ else } n^\iota = 0,$$

$$(k^z)' \geq 0, \quad c^z > 0,$$

$$\text{If } \Lambda^\iota = 0, \quad (\Lambda^\iota)' \in \{0,1\}, \text{ else } (\Lambda^\iota)' = \Lambda^\iota.$$

Since no marriages or divorces occur after age 65, we no longer need to keep track of value functions of individuals within couples. The value function of a single household, aged

65 and older, satisfies the Bellman equation:

$$V^S(k^z, e, u, a, \iota, \Lambda, j) = \max_{c^z,(k^z)',n,(\Lambda)'} \Big[U^S\left(c^z, n, \iota\right)$$

$$+ \beta E_{u'} \big[V^S((k^z)', e', u', a, \iota, (\Lambda)', j+1) \big] \tag{12}$$

where maximization is subject to

$$c^z(1+\tau_c) + (k^z)'(1+\mu) = k^z\left(1 + r(1-\tau_k)\right) + g^z + Y^L + \Psi^z \Lambda$$

$$Y^L = \left(Y^{L,\iota}\right)\left(1 - \tau_{ss} - \tau_l^S\left(Y^{L,\iota}\right)\right)$$

$$Y^{L,\iota} = \frac{n^\iota w^{z,\iota}\left(a^\iota, e^\iota, u^\iota\right)}{1+\tilde{\tau}_{ss}}, \ \iota = m, w$$

$$(e^\iota)' = e^\iota + \mathbb{1}_{[n^\iota > 0]},$$

$$\text{If } \Lambda = 0, \quad n^\iota \in [0,1], \ \text{else } n^\iota = 0,$$

$$(k^z)' \geq 0, \quad c^z > 0,$$

$$\text{If } \Lambda = 0, \quad (\Lambda)' \in \{0,1\}, \ \text{else } (\Lambda)' = \Lambda.$$

2.7 Competitive Equilibrium

Appendix A.1 defines a competitive equilibrium. We focus on a stationary equilibrium in which price-taking agents optimize, markets clear, budgets balance, and the cross-section distribution across household types is time invariant.[14].

3 Calibration

We calibrate parameters to match selected moments from 2001-2007 U.S. data. We calibrate parameters listed in Table 1 directly to their empirical counterparts; we don't need to use our model to calibrate them. We calibrated the 11 parameters in Table 2 using an exactly identified simulated method of moments (SMM) approach.

[14] The associated BGP can of course be constructed by scaling all growing variables by the factor Z_t.

3.1 Technology

We set the capital share parameter α to 1/3 and choose the depreciation rate to match an investment-to-capital ratio of 9.88% in U.S. data.

3.2 Demographics and Transition Between Family Types

The demographic structure of the model is determined by the unit mass of newborn households and the death probabilities of retirees. We obtain the latter from the National Center for Health Statistics.

There are three family types: (1) single males; (2) single females; (3) married couples. To calculate age-dependent probabilities of transitions between married and single, we use U.S. data from the CPS March supplement, covering years 1999 to 2001. We assume stationarity; thus, although we allow probabilities of transitioning between family types to depend on an individual's age, we rule out dependence on birth cohort. We compute the probability $M(j)$ of getting married and the probability $D(j)$ of getting divorced at age j from the transition equations:

$$\bar{M}(j+1) = (1-\bar{M}(j))M(j) + \bar{M}(j)(1-D(j)),$$
$$\bar{D}(j+1) = \bar{D}(j)(1-M(j)) + \bar{M}(j)D(j).$$

Exogenous spousal sorting by ability is governed by the parameter φ that we estimate with our SMM procedure that makes the model match the empirical correlation of hourly wages of 0.407 in the CPS (2001-2007) for married couples.[15]

[15]Specifically, prior to marriage an individual of earnings type a draws random marriage quality $\varsigma \sim U[0,1]$. His/her marriage quality rank M_n is then determined by

$$M_n = (1-\varphi)\varsigma + \varphi a. \tag{13}$$

Then all individuals of the same gender are ranked according to M_n and matched with exactly the same rank of the opposite gender. If $\varphi = 0$, marriage is random, and if $\varphi = 1$, marriages are perfectly sorted by spousal ability a. Appendix A.5 contains the details of this construction, which, conditional on own ability a, induces a distribution over spousal abilities (and associated distribution over the other payoff-relevant state variables of future partners) that permits singles to rationally form expectations.

3.3 Wages

We estimate experience profiles for male and female wages and the exogenous processes for idiosyncratic shocks using the PSID from 1968-1997. After 1997, it is not possible to obtain years of actual labor market experience from the PSID. Appendix A.4 describes our estimation procedure in more detail. We use a 2-step approach to control for selection into the labor market, as described in Heckman (1976, 1979). After estimating returns to experience for males and females, we use residuals from the regressions and the panel data structure of the PSID to estimate parameters ρ_ϵ^ι and σ_ϵ^ι for the productivity shock processes and the variances σ_a^ι of individual abilities. We estimate the mean wage parameters γ_0^w and γ_0^m together with other model parameters via SMM. The associated data moments are the ratio between male and female earnings and the average wage of working individuals, which we normalize to 1 in the model.

We set the parameter ϕ_1 that controls the speed of the human capital depreciation to one used by Graves et al. (2023); we set ϕ_2 that controls when the human capital depreciation starts to depreciate to 66.

3.4 Preferences

One-period utility functions for both family types are given in equations (5) and (6). We set the discount factor β to match the capital-output ratio K/Y, taken from the BEA. We choose four different values for the participation costs, by gender and marital state, to match employment rates of married and single males and females aged 20-64, taken from the CPS.

As explained by Keane (2011), economists disagree about sizes of Frisch elasticities of labor supply. Nevertheless, there seems to be a consensus that female labor supply is much more elastic than male labor supply.[16] We set the intensive margin Frisch elasticity of male labor supply $1/\eta^m = 0.4$, in line with recent work in quantitative macroeconomics; see Guner et al. (2012). We set the intensive margin Frisch elasticity of female labor supply $1/\eta^w$ to

[16] The recent paper by Blundell et al. (2016) estimates the intensive margin Frisch elasticity for men and women between the ages of 30 and 57 as 0.53 and 0.85, respectively.

0.8.

3.5 Taxes and Social Security

As described in Section 2.5 we employ the Benabou (2002) labor income tax function $\tau(y) = 1 - \theta_0 y^{-\theta_1}$. We follow HKS and use U.S. labor income tax data provided by the OECD to estimate the parameters θ_0 and θ_1 for different family types.

For the government-run social security system we assume that payroll taxes for the employee, τ_{SS}, and the employer, $\tilde{\tau}_{SS}$ are flat taxes, and use the rate from the bracket covering most incomes in the U.S., 7.65% for both τ_{SS} and $\tilde{\tau}_{SS}$. We follow Trabandt & Uhlig (2011) and set $\tau_k = 36\%$ and $\tau_c = 5\%$ for consumption and capital income tax rates.

3.6 Pensions

Sizes of pension payments depend on an individual's gender, marital status, fixed ability type, and accumulated experience:

$$\Psi(\iota, q, a_i, e_i) = \psi_0 + \psi_1 \times AE(\iota, q, a_i) \times \min\{1, e_i/35\}$$

To calibrate parameters ψ_0 and ψ_1, we use three data moments obtained from the OECD publication *Pensions at a glance*, (2007): i) the average replacement rate across all individuals is 40%, ii) the average pension of someone with income of 0.5 times average earnings (0.5AE) is 0.276AE, iii) the average pension of someone with income of 2.0 times average earnings (2.0AE) is 0.643AE. We use this information to pin down $\psi_0 = 0.1537$, $\psi_1 = 0.2447$.

Before the social reform that we shall describe below, if an individual 65 or older decides to work, he or she cannot collect pension payments, a feature that imposes an implicit tax on working and can induce some people to retire via a corner solution to their choice problem. We will study the effects on the aggregate labor supply elasticity of reforms that remove this implicit tax.

Table 1: Parameters Calibrated Outside of the Model

Parameter	Value	Description	Target
$1/\eta^m, 1/\eta^w$	0.4, 0.8	$U^M(c, n^m, n^w) = \log(c) - \chi_M^m \frac{(n^m)^{1+\eta^m}}{1+\eta^m} - \chi_M^w \frac{(n^w)^{1+\eta^w}}{1+\eta^w} - F_M^w \cdot \mathbb{1}_{[n^w>0]}$	Literature
$\gamma_1^m, \gamma_2^m, \gamma_3^m$	$0.061, -1.06 * 10^{-3}, 9.30 * 10^{-6}$	$w_t(a_i, e_i, u_i) = w_t e^{a_i+\gamma_0^m+\gamma_1^m e_i+\gamma_2^m e_i^2+\gamma_3^m e_i^3+u_i}$	PSID (1968-1997)
$\gamma_1^w, \gamma_2^w, \gamma_3^w$	$0.078, -2.56 * 10^{-3}, 2.56 * 10^{-5}$	$w_t(a_i, e_i, u_i) = w_t e^{a_i+\gamma_0^w+\gamma_1^w e_i+\gamma_2^w e_i^2+\gamma_3^w e_i^3+u_i}$	
$\sigma_\epsilon^m, \sigma_\epsilon^w,$	0.322, 0.310	$u' = \rho_{jg} u + \epsilon$	
$\rho_\epsilon^m, \rho_\epsilon^w$	0.396, 0.339	$\epsilon \sim N(0, \sigma_{jg}^2)$	
σ_a^m, σ_a^w	0.315, 0.385	$a^\iota \sim N(0, \sigma_{a_m}^2)$	
$\theta_0^S, \theta_1^S, \theta_0^M, \theta_1^M$	0.8177, 0.1106, 0.9420, 0.1577	$ya = \theta_0 y^{1-\theta_1}$	OECD tax data
τ_k	0.36	Capital tax	Trabandt & Uhlig (2011)
$\tau_{ss}, \tilde{\tau}_{ss}$	(0.0765, 0.0765)	Social Security tax	OECD
τ_c	0.05	Consumption tax	Trabandt & Uhlig (2011)
T	$0.2018 \cdot AW$	Income if not working	CEX 2001-2007
g/Y	1%	Transfers to households	2X military spending (World Bank)
B/Y	0.6185	National debt	Government debt (World Bank)
$\omega(j)$	Varies	Survival probabilities	NCHS
$M(j)$, $D(j)$	Varies	Marriage and divorce probabilities	CPS
μ	0.0200	Output growth rate	Trabandt & Uhlig (2011)
δ	0.0788	Depreciation rate	$I/K - \mu$ (BEA)
α	1/3	$Y_t(K_t, L_t) = K_t^\alpha [Z_t L_t]^{1-\alpha}$	Historical capital share
ϕ_1	0.2	speed of human capital depreciation in old age	Graves et al. (2023)
ϕ_2	66	age when depreciation of human capital starts	
ψ_0	0.1537	minimum pension benefit	"Pensions at a glance" (2007)
ψ_1	0.2447	increase in pension benefits with lifetime earnings	

Table 2: Parameters Calibrated Endogenously

Parameter	Value	Description	Moment	Moment Value
γ_0^m	0.070	$w_t(a_i, e_i, u_i) = w_t e^{a_i+\gamma_0^\iota+\gamma_1^\iota e_i+\gamma_2^\iota e_i^2+\gamma_3^\iota e_i^3+u_i}$	Gender earnings ratio	1.569
γ_0^f	-0.066		Average Earnings (AE)	1.000
β	1.007	Discount factor	K/Y	2.640
$\log(F_M^w)$	-1.504	$U^M(c, n^m, n^w) = \log(c) - \chi_M^m \frac{(n^m)^{1+\eta^m}}{1+\eta^m} -$	Married fem employment	0.676
χ_M^w	5.96	$\chi_M^w \frac{(n^w)^{1+\eta^w}}{1+\eta^w} - F_M^m \cdot \mathbb{1}_{[n^m>0]} - F_M^w \cdot \mathbb{1}_{[n^w>0]}$	Married female hours	0.224 (1225 h/year)
$\log(F_M^m)$	-0.506		Married male employment	0.879
χ_M^m	17.11		Married male hours	0.360 (1965 h/year)
$\log(F_S^w)$	-0.086	$U^S(c, n, \iota) = \log(c) - \chi_S^\iota \frac{(n)^{1+\eta^\iota}}{1+\eta^\iota}$	Single fem. employment	0.760
χ_S^w	12.93	$-F_S^\iota \cdot \mathbb{1}_{[n>0]}$	Single female hours	0.251 (1371 h/year)
$\log(F_S^m)$	0.308		Single male employment	0.799
χ_S^m	39.95		Single male hours	0.282 (1533 h/year)
φ	0.1229	$M_n = (1-\varphi)\varsigma + \varphi a$	$corr(a^m, a^w)$	0.646

3.7 Transfers and Government Consumption

There is an ongoing debate about the share of government spending that transfers to households comprise. Here we assume that most government spending is on public goods. In the calibrated equilibrium we give all individuals the same small lumpsum transfer that totals 1% of GDP. All other government expenditures are on the pure public consumption good G.

3.8 Estimation Method

Twelve model parameters are estimated using an exactly identified simulated method of moments. We minimize the squared percentage deviation between simulated model statistics and the twelve data moments in column 5 of Table 2. Let $\Theta = \{\gamma_0^m, \gamma_0^f, \beta, F_M^w, \chi_M^w, F_M^m, \chi_M^m, F_S^w, \chi_S^w, F_S^m, \chi_S^m, \varphi\}$, and let $V(\Theta) = (V_1(\Theta), \ldots, V_{16}(\Theta))'$ and $V_i(\Theta) = (\bar{m}_i - \hat{m}_i(\Theta))/\bar{m}_i$, which measures a percentage difference between empirical and simulated moments. We set Θ to the minimizer of $V(\Theta)'V(\Theta)$. Table 2 summarizes estimated parameter values and data moments. We match all moments exactly, so $V(\Theta)'V(\Theta) = 0$.

4 Mechanics

4.1 Sources of individual fortunes

Exogenous stochastic marital dynamics, including assortative matching in terms of ability types, shape an individual's lifetime utility. Making marital dynamics exogenous helps us

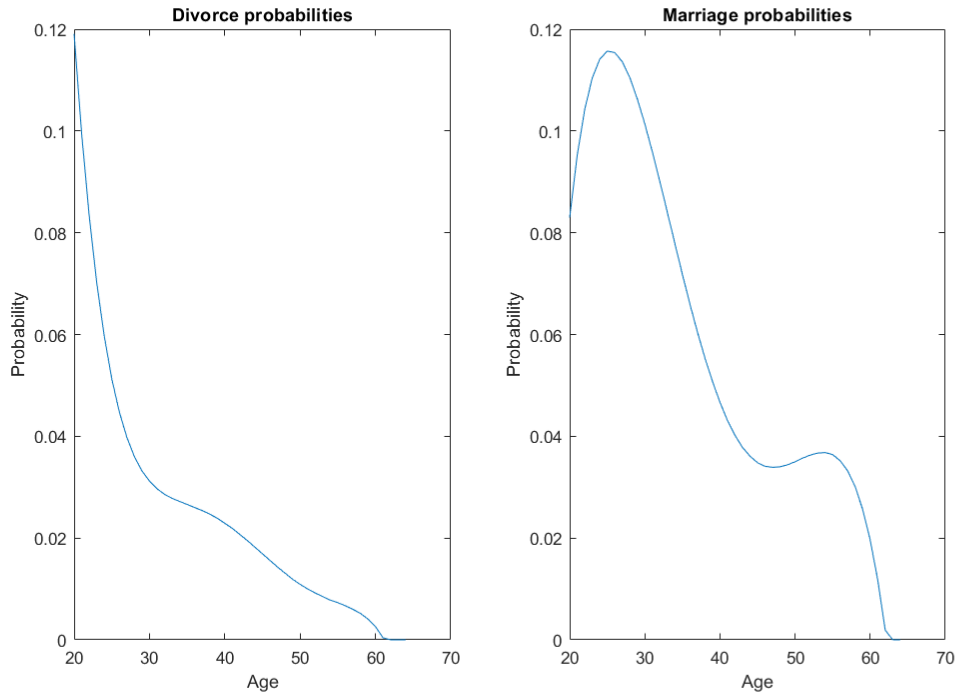


Figure 1: Divorce and marriage probabilities by age

isolate and highlight how time averaging determines individuals' and couples' decisions about labor supply, consumption, and savings.

Spouses are the same ages and die simultaneously. Figure 1 depicts marriage and divorce probabilities at different ages. Figure 2 shows fractions of singles at different ages. After age 65, the fraction of singles remain constant since divorces and new marriages stop.

An individual's gender affects its lifetime utility. A single individual has a gender-specific disutility of working, while a married couple shares the same utility function over the couple's consumption and the couple's labor supplies. In addition to gender-specific permanent ability types and the stochastic process for idiosyncratic wage shocks, the wage of an individual depends on a gender-specific labor market experience profile displayed in Figure 3.

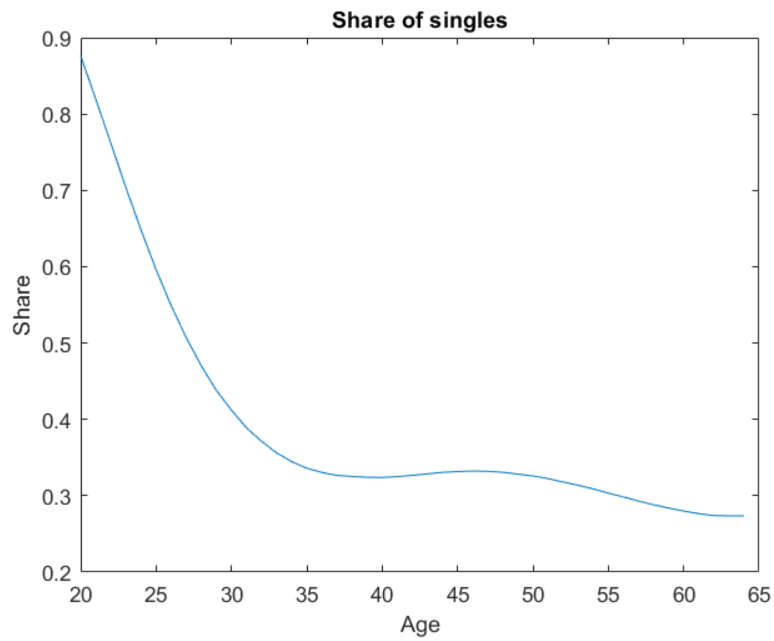


Figure 2: Share of singles by age

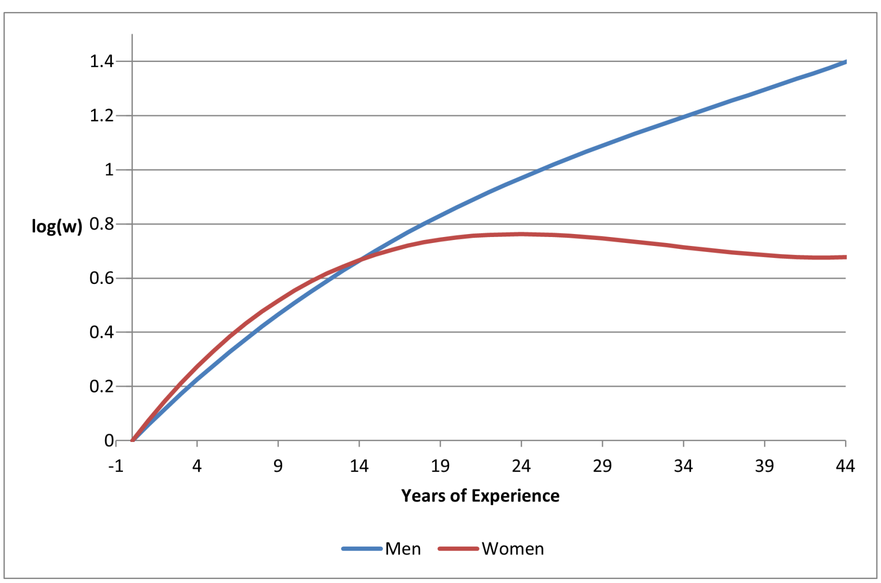


Figure 3: Labor market experience component of wages by gender and years of experience

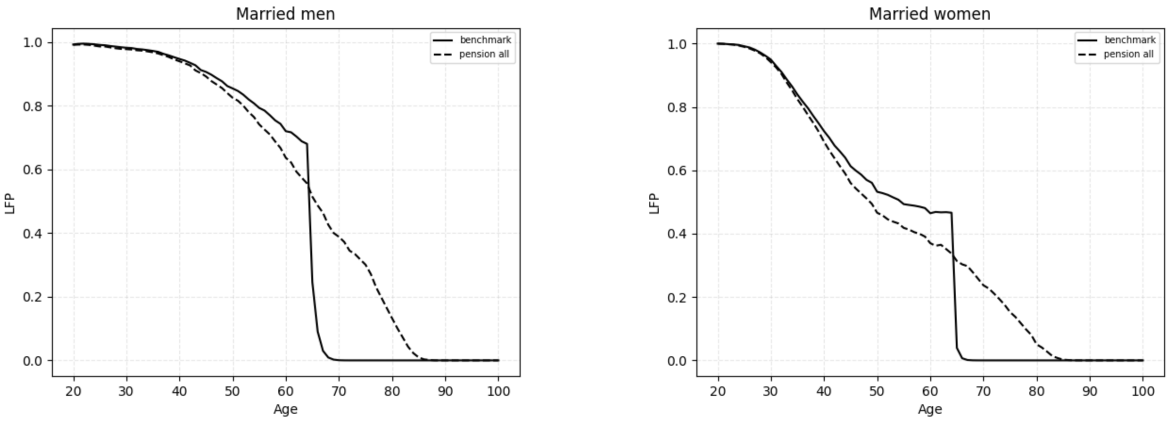


Figure 4: Labor force participation in benchmark economy and under social security reform by married men (Panel A) and married women (Panel B)

Subsections 4.2 thru 4.4 will soon be revised. For now, they temporarily report outcomes under the HKS recursive formulation of households' optimization problems described in footnote 10. These computations are being updated under the reformulation described in that footnote.

4.2 Benchmark economy

As indicated by the solid lines in Figures 4 and 5, our time-averaging version of HKS can reproduce the outcome of HKS that people retire by age 65. The main features that let our model attain this outcome are parameterizations of fixed disutilities of working and a social security system that mimics an earlier system of the U.S. that specificied that working beyond age 65 meant that some benefits would be lost.[17] In subsection 4.3, a reform allows individuals to collect their benefits from age 65 whether or not they retire. The composition of earners in marriages of different ages is shown in panel A of Figure 6; either both spouses work, only the man or the woman works, or neither spouse works.

[17] As for our assumption that social security benefits not collected after age 65 cannot be recovered later, Schulz (2001, pp. 141-2) describes how this was the situation in the U.S. social security system between 1950 and 1972, after the repeal in 1950 of an earlier provision of a 1 percent increase in benefits for each year of delayed retirement. After 1972, a delayed retirement credit was reintroduced, but it is only with rules that recently became effective that the compensation is high enough for there to be no loss in the actuarial value of a worker's lifetime benefits.

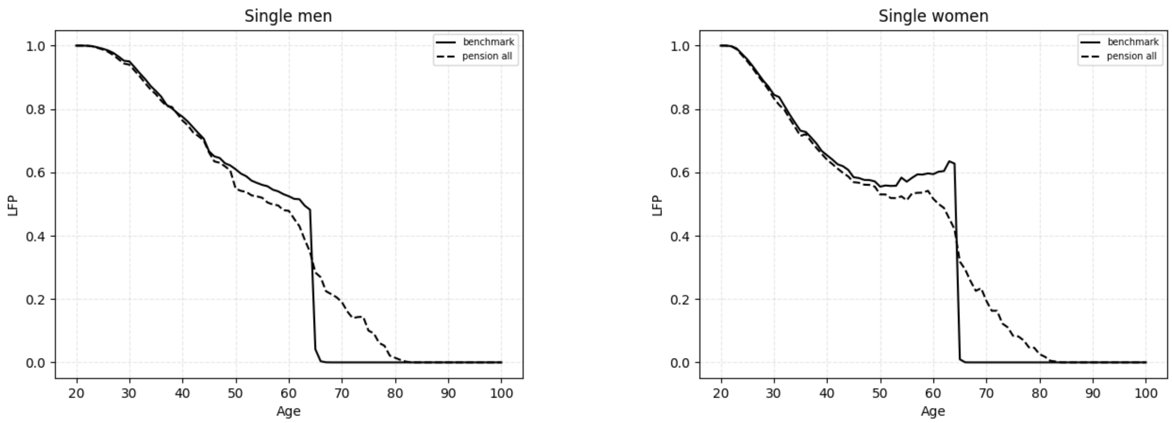


Figure 5: Labor force participation in benchmark economy and under social security reform by single men (Panel A) and single women (Panel B)

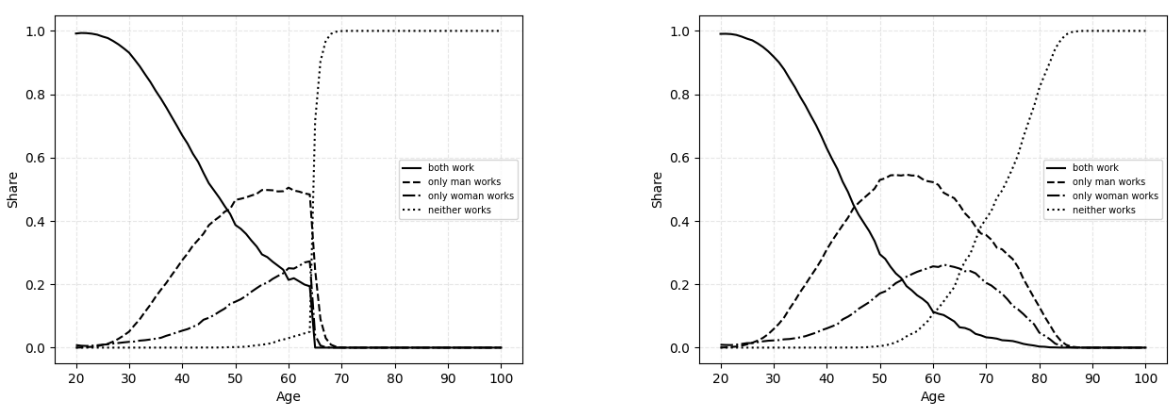


Figure 6: Composition of married couples' labor supply in benchmark economy (Panel A) and under social security reform (Panel B)

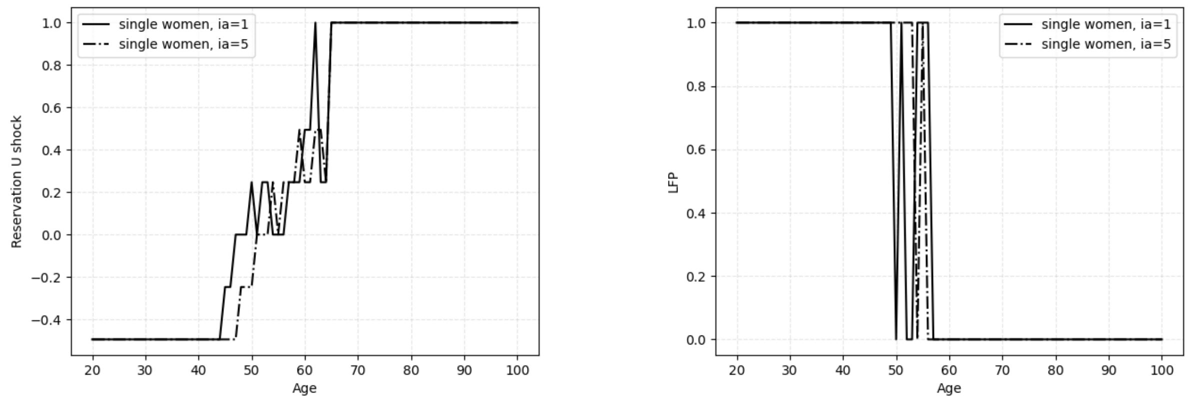


Figure 7: Reservation productivity (Panel A) and labor force participation (Panel B) of a single woman of the lowest and highest ability, respectively, who is unknowingly experiencing a life of fixity with respect to singlehood and an idiosyncratic productivity shock equal to its median value of $log(1) = 0$.

In addition to gender, age, and permanent ability, decision rules are functions of marital state, years of labor market experience, and asset holdings. To illustrate how consumption, savings, and labor supplies evolve over the life cycle in our time-averaging model, we devise the following thought experiment. We study a special agent who ends up with a time invariant marital state and time invariant idiosyncratic productivity shock.[18]

To study agents who fare especially poorly, we focus on a single woman of the lowest ability type who experiences a life of singlehood and has an idiosyncratic productivity shock always equal to its median value of $log(1) = 0$. The solid lines in Figure 7 depict her choice of reservation productivity at each age in panel A (where a value of 1 means that no emploment opportunity is acceptable) and the resulting labor force participation in panel B, either to work (value 1) or not to work (value 0). The solid lines in Figure 8 depict her associated decisions to consume in panel A and how many assets to hold in panel B. As a contrast, we also show that same thought experiment for a single woman of the highest ability type (dashed lines) in Figures 7 and 8.

From comparing a single woman of the lowest ability to a one of the highest ability, we see

[18] In constructing the section 2 decision rules, the agent was of course planning for other possible outcome paths too.

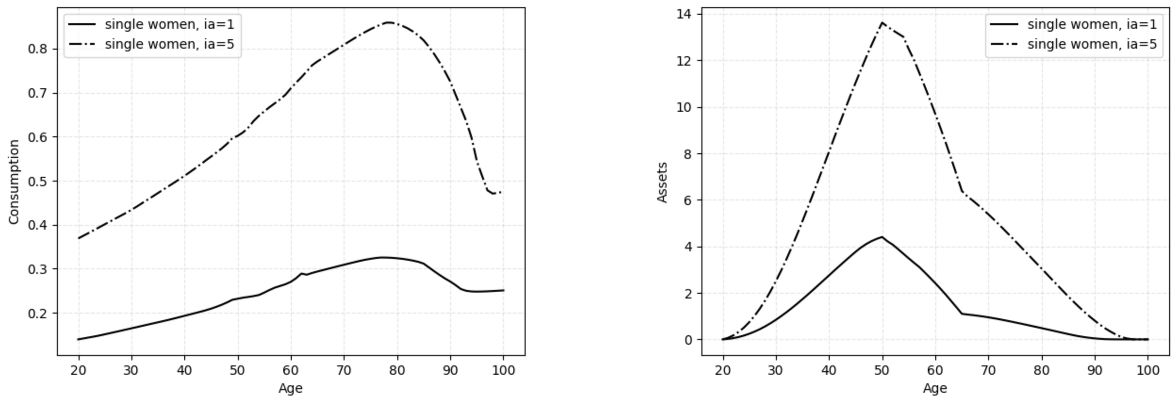


Figure 8: Consumption (Panel A) and asset holdings (Panel B) of a single woman of the lowest and highest ability, respectively, who is unknowingly experiencing a life of fixity with respect to singlehood and an idiosyncratic productivity shock equal to its median value of $log(1) = 0$.

a couple of striking similarities. One is that both women stop working at age 65, since they both set the reservation productivity equal to 1 in panel A of Figure 7, which exceeds the highest productivity shock (and that we use to indicate that 'no employment opportunity is acceptable'). This outcome is not very surprising since we parameterized the model to make most agents retire by age 65: rules of the social security system put many agents at a corner solution that tells them to retire at age 65. More surprising is another similarity: in spite of very different consumption and asset holdings in Figure 8, both women have similar reservation productivities throughout their working lives. An explanation for those similar choices is threefold: (i) different abilities manifest as multiplicative shifts in an otherwise common woman-specific labor market experience profile in Figure 3; (ii) the social security benefit is scaled by an indicator of an agent's past labor earnings, so benefits are similar multiples of past labor earnings for both women; and (iii) the utility function is consistent with balanced growth. S uch a utility function makes the level of earnings *per se* irrelevant so long as other relevant conditions are equivalent, such as similar elasticities of earnings to accumulated working time and a similar replacement rate in the social security system.[19]

[19] See footnote 5 for how the elasticity of earnings to accumulated working time affects the career length.

The chattering of reservation productivities and consequently in labor market participations at the end of women's working lives portrayed in Figure 7 is a natural outcome in a stochastic time-averaging model when a person wants to sustain optimal precautionary and/or retirement savings. The individual's labor supply responds both to favorable opportunities and to motives to replenish savings after voluntary episodes of not working have reduced her asset holdings.

4.3 How tax revenues are spent matters

In an employment-lottery framework, Prescott (2002) emphasized that effects of taxation depend on whether a government spends tax revenues on close substitutes for private consumption or on public goods that are not close substitutes. Here we conduct a corresponding analysis in our time-averaging model. Our approach is two-prongerd. First, we import our Section 5 insights from studying outcomes under various tax reforms. Second, we compare outcomes with models that fully or only partially incorporate time-averaging.

Prescott said that most tax revenues are spent for goods and services that substitute perfectly for private consumption. He modelled government expenditures as lump-sum transfers to households. Temporarily embracing Prescott's assumption, we explore effects of the following tax and tranfer scheme in our benchmark model. While keeping our present system of taxation, we introduce an additional flat-rate tax $\bar{\tau}$ that is levied on what had been agents' after-tax labor income. Thus, if that new flat-rate tax is set equal to zero, we are back to the earlier equilibrium of our benchmark economy; if that tax rate is set equal to 100 percent, agents keep nothing of their labor incomes. We consider two alternatives for how the government uses the revenues collected from levying the new tax rate: they are either handed back as an equal lump-sum transfer to all agents in the economy, or spent on our pure public good G. In both cases, we adopt the auxiliary assumption that social security benefits are also reduced by the percentage of the new flat-rate tax.

Figure 9 shows tax revenues raised by levying the new tax $\bar{\tau}$ when tax revenues are handed back as lump-sum transfers (solid line) or spent on public goods (dashed line).[20] Much smaller tax revenues are collected along the solid Laffer curve because the lump-sum transfer cancels the income effect of a higher tax rate $\bar{\tau}$. That gives the substitution effect full rein to reduce individuals' labor supplies. These two Laffer curves reconfirm Prescott's (2002, p. 7) assertion that "the assumption that the tax revenues are given back to households either as transfers or as goods and services [that are good substitutes to private expenditures] matters. If these revenues are used for some public good or are squandered, private consumption will fall, and the tax wedge will have little consequence for labor supply." Nevertheless, because capital formation is affected in a general equilibrium, the additional tax wedge $\bar{\tau}$ brings distortions that increase along with the tax rate. To examine how much of the distortions operate through capital formation, the dotted line in Figure 9 is the Laffer curve for a small open-economy version of the tax experiment without lump-sum rebates when the interest rate is held constant at the benchmark equilibrium rate.

To illustrate dismal labor supplies accompanying the solid Laffer curve, Figure 10 shows labor force participations of single women by ability types relative to benchmark model outcomes as functions of the additional tax $\bar{\tau}$. We can also imagine how this tax and transfer program would undo the outcome of similarities in our Section 4.2 thought experiment that followed single women of lowest ability and highest ability who shared the same realizations of productivity shocks and marital state. In that experiment, choices of reservation productivites and associated labor force participations were similar across the two women. In contrast, Figure 10 shows that single women of the lowest ability are the first to withdraw from labor market participation, while participation of the highest ability is more resilient to

[20] Note that the tax revenues from levying the new tax $\bar{\tau}$ are in general higher than the change in total revenues from labor income taxation. Namely, smaller tax revenues can be expected to be raised from the benchmark taxation of labor income whenever a higher tax rate $\bar{\tau}$ causes labor supplies and consequently aggregate labor income to shrink. Such losses of benchmark tax revenues do not affect our calculation of a lump-sum transfer. Our tax experiment risks bankrupting the government, so as we raise the tax $\bar{\tau}$, we must verify that the sum of labor income tax revenues not handed back as lump-sum transfers and the revenues from the capital tax and the payroll (social security) tax are sufficient for the government to finance social security benefits, interest on government debt, and the original small per-capita lump-sum transfer g totalling 1 percent of GDP in the benchmark economy.

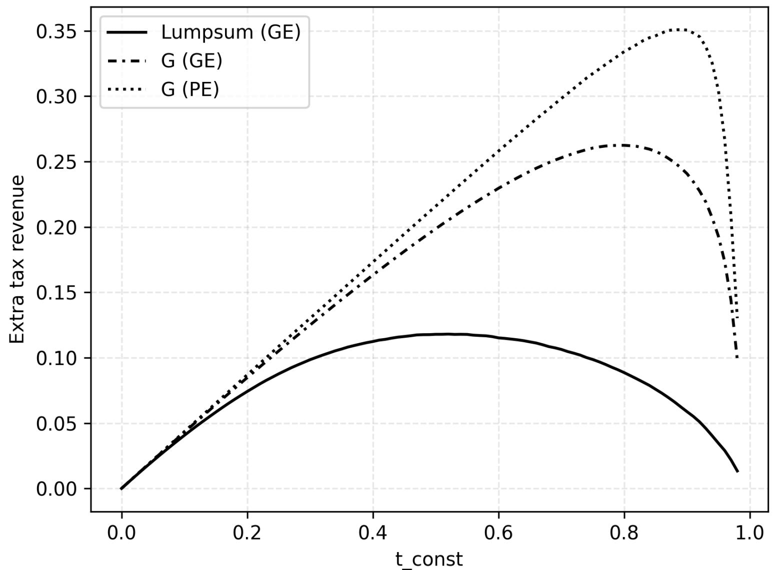


Figure 9: Laffer curves

the distortions brought by the new tax and transfer scheme. Because the lump-sum transfer as a fraction of potential labor earnings differs so much across ability levels, so does the size of the associated income effect.

To emphasize how important it is whether a model fully implements time-averaging, we now compare outcomes across our benchmark model and the original HKS model. We revisit two of HKS's policy settings: the U.S. tax system and a flat tax system. For each system we compute the peak of the Laffer curve (as a percent of benchmark tax revenue) for the two alternative uses of new tax revenues: either handed back as a lump-sum transfer, or spent on the pure public good G. Table 3 brings out how difference in outcomes between the two uses of tax revenues are big in our full-fledged time-averaging model and much smaller in the HKS model that implemented time-averaging only partially.

4.4 Benchmark economy under social security reform

From here on, the retirement system contains no implicit tax on working beyond age 65. Whether they work or not, at age 65 agents start to receive social security benefits according

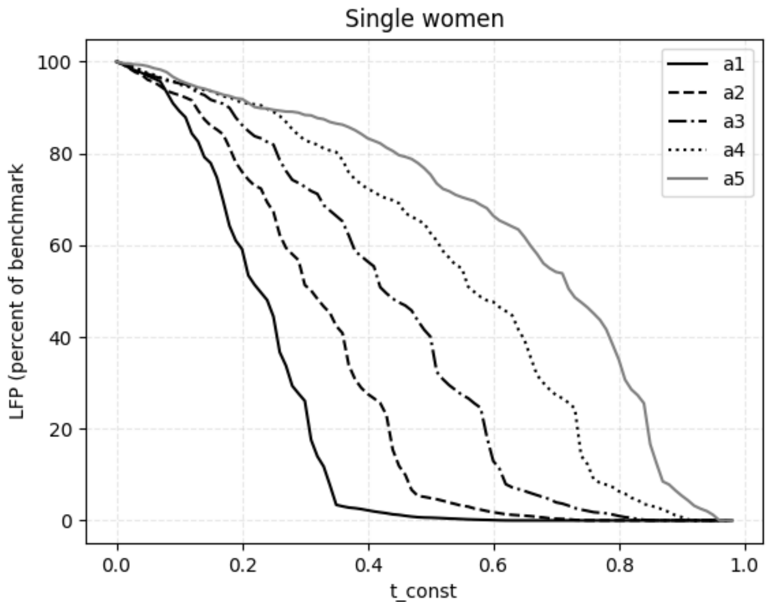
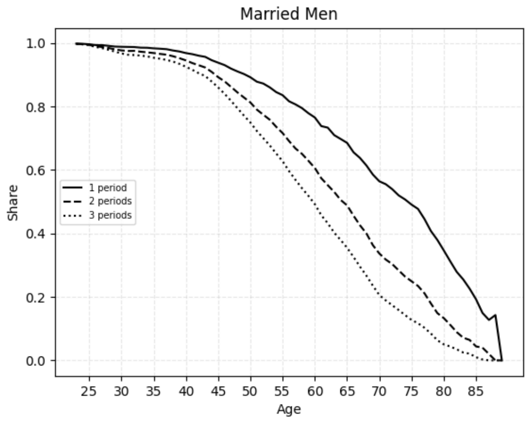


Figure 10: Labor force participation of single women by ability type relative to benchmark outcome as a function of the additional tax $\bar{\tau}$

Table 3: The Peak of the Laffer Curve for Different Uses of Revenue

Benchmark model			
	G	Lumpsum	Lumpsum/G
Flat Tax	193.3	148.8	77.0
U.S. Tax System	176.8	144.2	81.6
HKS model			
	G	Lumpsum	Lumpsum/G
Flat Tax	179.4	170.0	94.7
U.S. Tax System	167.1	159.3	95.3

The Table displays the peak of the Laffer curve (in percent of benchmark tax revenues) for a flat tax and with the current U.S. tax system, under two different assumptions about the use of new tax revenues. "G" denotes that new tax revenues are used to finance pure public goods, while "Lumpsum" denotes that the revenues are handed back as lump-sum transfers to the households.



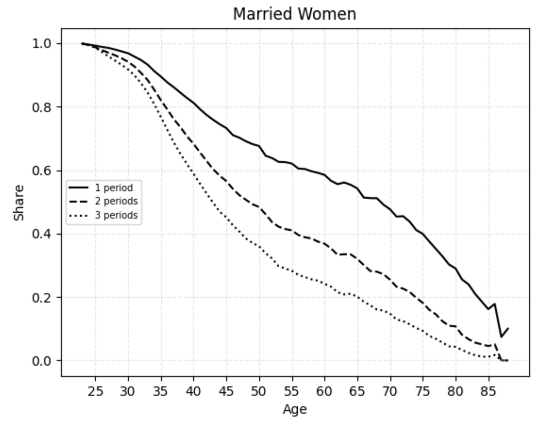


Figure 11: Persistence of employment in benchmark economy under social security reform by married men (Panel A) and married women (Panel B). A curve labelled "x period(s)" shows the fraction of presently working agents who were also working in the previous x periods, for $x = 1, 2, 3$, as a function of age.

to formula (7).

Our having parameterized a benchmark economy under an earlier social security system serves two purposes. First, we confirm that we can make most people choose to retire by age 65 by specifying appropriate social security benefit rules. Second, our approach helps us to discipline parameterizations of disutilities in our time-averaging model when retirement patterns among older workers were gradually changing as they did in the U.S. during the recent transition to a system with actuarially fair benefit calculations for those who chose to delay receiving social security benefits. When we disarm the corner solution in the benchmark model by installing our assumed social security reform, the model offers predictions about equilibrium outcomes after the reform.

Those predictions can be read off from the dashed lines in Figures 4 and 5 as well as from panel B in Figure 6. In addition, here we compute the persistence of employment spells as functions of age, as shown for married individuals in Figure 11 and for single individuals in Figure 12. A curve labelled "x period(s)" shows fractions of presently working agents who were also working in x previous periods, for $x = 1, 2, 3$, as a function of age. Outcomes are reported when at least 0.1 percent of agents are working at a given age.

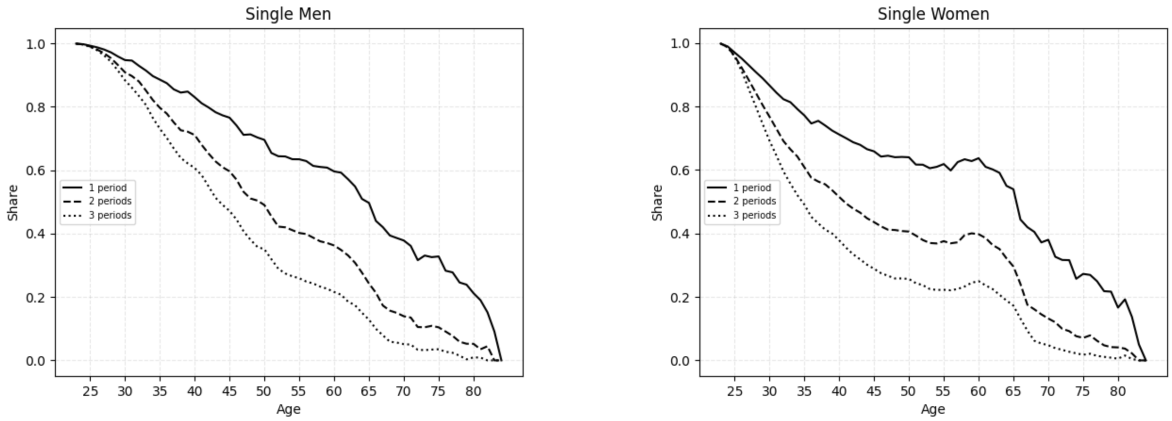


Figure 12: Persistence of employment in benchmark economy under social security reform by single men (Panel A) and single women (Panel B). A curve labelled "x period(s)" shows the fraction of presently working agents who were also working in the previous x periods, for $x = 1, 2, 3$, as a function of age.

4.5 Ex post-ex ante welfare measure

In addition to standard *ex ante* welfare measures, either unconditional or conditional on characteristics such as gender and ability type, we also construct what we call *ex post-ex ante* welfare measures that respond to concerns about some bad states that some individuals visit. A challenge here is that the identities of such individuals change over time. What does it mean to favor policies that help such classes of people whose members change over time? We propose a welfare measure that includes all agents who *ex post* experience that state at a given age. We compute each such agent's experienced utility up and until that age and, as a continuation value, evaluate that agent's value function at his/her state at that age, then discount utilities back to the time when the person entered the economy. Next, by averaging over all those computed lifetime utilities in the subpopulation of agents who experience that state at that age, we arrive at our *ex post-ex ante* welfare measure.

To gauge how inequality is related to marital state, let's put this *ex post-ex ante* welfare measure to use in our benchmark economy under a social security reform Specifically, we compute these welfare measures for each gender for someone who *ex post* experienced a particular marital state at a given age. Then we express welfare as consumption equivalents

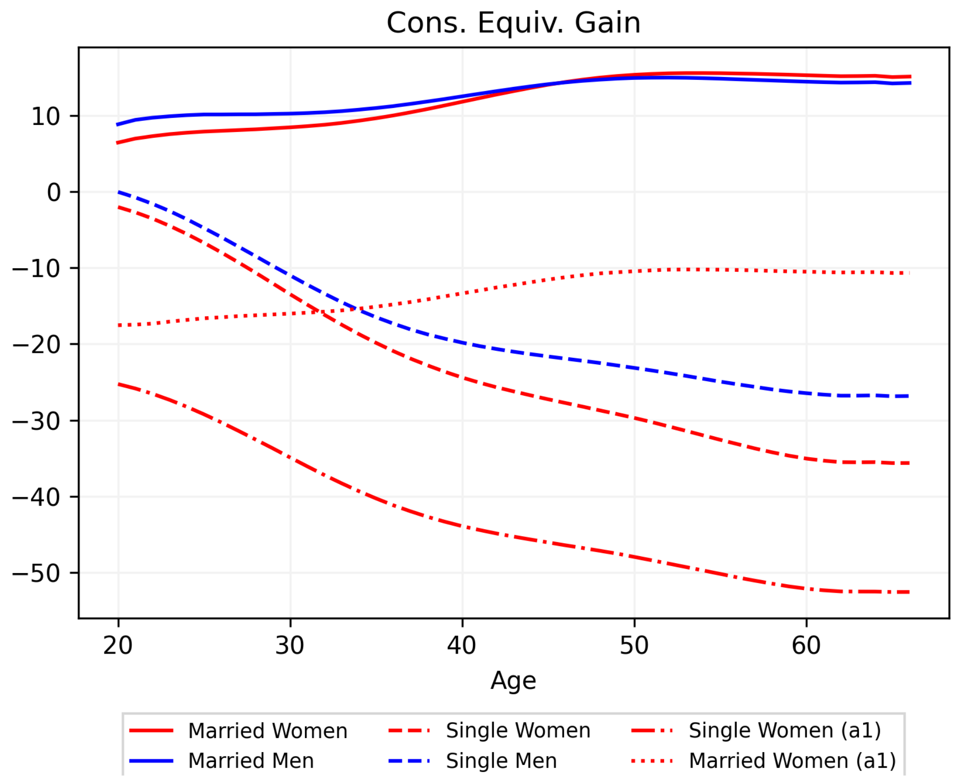


Figure 13: Ex post-ex ante welfare measures by marital state and gender as a function of age in the benchmark economy under social security reform. The dotted and dash-dotted lines break out females of the lowest ability as married and single, respectively. The consumption equivalents are relative to the unconditional lifetime utility of a 20-year old entering the economy under the veil of complete ignorance.

relative to the unconditional lifetime utility of an agent put into the economy as a 20-year old (the age at which a new cohort enters the economy) under the veil of complete ignorance about gender, ability type, marital state, and initial idiosyncratic productivity shock. Those *ex post-ex ante* welfare measure are reported in Figure 13.

The four categories of males and females who are either married or single in Figure 13 make up an entire cohort that we trace over time as the cohort ages until age 65, after which the marital state is constant. The figure reveals that singlehood is less advantageous than marriage. Starting at age 20, a small group of agents is already married; these people have the highest welfare as measured by a standard *ex ante* welfare measure conditional on being married for each gender (this which coincides with our *ex post-ex ante* welfare measure at age 20). However, most people are single at age 20; since they constitute most of the population, their consumption equivalents are close to zero. For two reasons, singlehood becomes more and more disadvantageous as people age. Our *ex post-ex ante* welfare measure is both backward- and forward-looking. If someone is single at a ripe old age, the low marriage probabilities in Figure 1 make the future look bleak; and the recent past was most likely also spent in singlehood. This twofold misfortune makes the *ex post-ex ante* consumption equivalent for the state of singlehood fall. Low divorce probabilities in old age make things work in the opposite direction for married people; for them, *ex post-ex ante* welfare increases with age.[21]

Figure 13 also depicts the *ex post-ex ante* welfare measures conditional on being a female of the lowest ability type. Evidently, considerable heterogeneity is embedded within measures based exclusively on gender and marital state.

5 Tax reforms

From hereon the "benchmark economy" operates under the subsection 4.4 social security reform. We use that baseline to evaluate efficiency and distributional consequences of three classic and controversial reform proposals: a flat-rate tax, a negative income tax, and a universal basic income. In so doing, we draw parallels between alternative tax reform designs and Prescott's (2002) alternative subsection 4.3 tax-and-transfer schemes. We shall learn that using time-averaging model rather than Prescott's employment-lottery framework generates sharper differences across households that affect efficiency-redistribution tradeoffs.

[21]To check our algorithm, we can calculate the *ex post-ex ante* welfare for the *ex post* outcome of surviving until some age $x \leq 65$, i.e., we are including the entire population since there is no mortality before age 65. Hence, by our algorithm of computing each agent's experienced utility until age x and using his/her value function as a continuation value, followed by averaging over all agents, we should arrive at the unconditional expected utility of someone who is randomly dropped down as a 20-year old in the economy under the veil of complete ignorance.

5.1 Flat-rate tax reform

For a given *deduction* of labor income exempt from taxation, expressed as a fraction of average income in the benchmark economy, we compute the flat-rate tax that raises the same total tax revenues as in the benchmark economy.

Descriptive statistics of flat-rate tax equilibria in Tables 4 and 6 reveal tradeoffs between efficiency and redistribution. As indicated by a standard unconditional *ex ante* measure of welfare, efficiency falls as the level of income exempt from taxation rises. While the lifetime utility of an agent put into the economy as a 20-year old under a veil of complete ignorance is higher under all three flat-tax reforms relative to the benchmark economy, welfare changes expressed in consumption equivalents are 2.85%, 2.65%, and 2.07% for flat-tax reforms with deductions of 0, 0.2, and 0.4, respectively.

To understand why unconditional *ex ante* welfare falls as the deduction rises, consider a flat-tax economy that is populated by agents all of whom work and have labor earnings that exceed the income level that is exempt from taxation. Such an economy would be isomorphic to another economy in which all labor earnings are subject to the flat tax and in which agents receive a lump-sum transfer from the government equal to the product of the flat tax and the level of income exempt from taxation in the first economy. Thus, the policy of the second economy is an example of the Prescott tax-and-transfer scheme in subsection 4.3. In light of the isomorphism between the two economies, the first economy with a flat tax and a level of income exempt from taxation has the same distortions and welfare losses as the second economy under the Prescott tax-and-transfer scheme.

In our heterogeneous-agent model, average welfare gains of flat-tax reforms take into account mixed fortunes conditioned on genders and permanent abilities, as shown in Table 6. Not surprisingly, individuals of high abilities gain most from replacing the progressive tax system in the benchmark economy with a flat tax. Their gains are eroded when the level of income exempt from taxation is increased, while higher deductions improve welfare for individuals of the lowest ability. It is noteworthy that the only negative consumption

Table 4: Summary Statistics from Benchmark and Flat Tax Reform Models

	Benchmark	Flat Tax with 0 Deduction	Flat Tax with 0.2 Deduction	Flat Tax with 0.4 Deduction	NIT (1)	NIT (2)
Capital	2.35	2.67	2.62	2.53	2.56	2.41
Total labor supply (all ages)	0.2002	0.2145	0.2118	0.2085	0.2058	0.1949
Labor supply of single males 65+	0.00164	0.0046	0.00408	0.00343	0.00443	0.00400
Labor supply of married males 65+	0.01076	0.0166	0.0150	0.0129	0.014766	0.01233
Labor supply of single females 65+	0.000307	0.0015	0.00127	0.00082	0.00166	0.00175
Labor supply of married females 65+	0.00140	0.00084	0.00078	0.000737	0.000871	0.000844
Single Male labor supply before 65	0.2820	0.2974	0.2945	0.2914	0.2761	0.2423
Married Male labor supply before 65	0.3599	0.3857	0.3822	0.3752	0.3767	0.3678
Single female labor supply before 65	0.2509	0.2621	0.2579	0.2588	0.2378	0.2062
Married female labor supply before 65	0.2242	0.2448	0.2412	0.2355	0.2436	0.2448
Average labor income tax rate	16.64%	12.16%	12.82%	13.87%	13.60%	15.84%
Labor income tax rate married	15.86%	12.16%	13.41%	15.26%	13.68%	16.86%
Labor income tax rate single men	19.34%	12.16%	11.85%	11.58%	13.68%	14.28%
Labor income tax rate single women	16.39%	12.16%	10.82%	9.29%	12.97%	11.50%

equivalents of flat-tax reforms in Table 6 pertain to individuals of the lowest abilities for both men and women. Furthermore, according to our *ex post-ex ante* welfare measures, those negative welfare outcomes relative to the benchmark economy befall married individuals of the lowest ability. This reflects how marital-state-specific tax functions lend an advantage to married couples in the form of lower effective average tax rates, especially at the lowest earnings levels where the effective average tax rate can even be negative.

An important inference to be drawn from Table 6 is that the negative consumption equivalents of married individuals of the lowest ability persist as the deduction is raised as high as 0.4. Thus, even when most, if not all, of the concerned households' labor earnings are exempt from taxation, these agents would prefer the benchmark arrangement. At the same time, higher deductions cause higher efficiency losses for the economy as a whole.

5.2 Negative income tax

Milton Friedman cast the negative income tax for the U.S. tax system. Here, we instead proceed schematically in an effort to distil essential ingredients of Friedman's proposal by placing it in the context of our flat-tax system with a zero deduction. We assume that workers are eligible for the negative income tax until age 65, after which they can receive social security benefits.

An individual who is not working receives a benefit B from the government which is reduced in a linear fashion if the individual starts to earn labor income. We want the effective tax rate on any labor earnings y to be less than 100%; Friedman often set it to 50%. In our flat-tax context, a beneficiary of a negative income tax faces a tax wedge equal to $(\tau + s)$, where τ is the flat tax that everyone pays and s is the additional tax rate paid on labor income until the agent has "lost" the entire benefit B, at which point the individual becomes a "regular" tax payer who owes the government the flat tax τ levied on his/her entire labor income. Thus, a household's disposable labor income prior to age 65 is

$$D(y) = \max\Big\{(1 - \tau - s)y + B, \quad (1 - \tau)y\Big\}.$$

Table 5: Welfare Changes (Percentage Changes in Consumption Equivalent Units) in Flat Tax Reforms Compared to Benchmark

	Ex ante 20-year olds	Ex post-ex ante 65-year olds married	Ex post-ex ante 65-year olds single
		Flat Tax with 0 Deduction	
Men (a1)	−0.944	−1.541	1.542
Men (a2)	1.010	0.550	3.829
Men (a3)	3.216	2.910	5.521
Men (a4)	5.546	5.359	7.791
Men (a5)	7.869	7.823	10.228
Women (a1)	−1.611	−1.177	−0.163
Women (a2)	0.240	0.572	1.436
Women (a3)	2.353	2.662	3.454
Women (a4)	4.606	4.861	5.685
Women (a5)	6.931	7.274	7.433
		Flat Tax with 0.2 Deduction	
Men (a1)	−0.346	−1.034	2.362
Men (a2)	1.097	0.499	4.309
Men (a3)	2.842	2.438	5.354
Men (a4)	4.753	4.479	7.054
Men (a5)	6.727	6.583	9.094
Women (a1)	−0.689	−0.487	1.572
Women (a2)	0.634	0.758	2.416
Women (a3)	2.218	2.385	3.697
Women (a4)	4.003	4.108	5.296
Women (a5)	5.921	6.118	6.700
		Flat Tax with 0.4 Deduction	
Men (a1)	0.139	−0.687	2.970
Men (a2)	0.998	0.257	4.483
Men (a3)	2.147	1.613	4.809
Men (a4)	3.478	3.082	6.124
Men (a5)	4.933	4.642	7.446
Women (a1)	−0.209	−0.522	3.215
Women (a2)	0.674	0.457	3.142
Women (a3)	1.748	1.544	4.216
Women (a4)	2.971	2.838	4.730
Women (a5)	4.352	4.311	5.375

Table 6: Welfare Changes (Percentage Changes in Consumption Equivalent Units) in Negative Income Tax Reforms Compared to Benchmark

	Ex ante 20-year olds	Ex post-ex ante 65-year olds married	Ex post-ex ante 65-year olds single
NIT (1): $\widehat{y}_S = 0.5$, $\widehat{y}_M = 1.0$, $B_S = 0.05$, $B_M = 0.1$, $s = 0.1$			
Men (a1)	−0.455	−1.172	2.563
Men (a2)	0.602	−0.054	3.948
Men (a3)	2.160	1.682	4.961
Men (a4)	4.106	3.861	6.711
Men (a5)	6.285	6.149	9.011
Women (a1)	−0.186	−0.656	4.448
Women (a2)	0.581	0.298	3.591
Women (a3)	1.929	1.785	4.339
Women (a4)	3.664	3.537	5.718
Women (a5)	5.675	5.671	6.827
NIT (2): $\widehat{y}_S = 1.0$, $\widehat{y}_M = 1.0$, $B_S = 0.1$, $B_M = 0.1$, $s = 0.1$			
Men (a1)	−0.131	−1.781	3.063
Men (a2)	−0.049	−1.147	3.784
Men (a3)	0.895	0.133	4.122
Men (a4)	2.310	1.740	5.337
Men (a5)	3.988	3.608	6.983
Women (a1)	1.135	−0.738	10.192
Women (a2)	0.767	−0.879	7.001
Women (a3)	1.272	0.228	5.484
Women (a4)	2.358	1.648	5.375
Women (a5)	3.818	3.485	5.712

Table 7: Labor Supply

	Benchmark	Flat Tax (0.0)[22]	Flat Tax (0.2)	Flat Tax (0.4)	NIT (1)[23]	NIT (2) [24]
Male	0.329	1.066	1.056	1.039	1.024	0.967
Single male	0.282	1.054	1.044	1.033	0.979	0.859
Single male (a1)	0.270	1.053	1.038	1.024	0.922	0.668
Single male (a2)	0.274	1.068	1.059	1.054	0.971	0.820
Single male (a3)	0.284	1.047	1.040	1.030	0.982	0.880
Single male (a4)	0.289	1.052	1.041	1.028	0.997	0.922
Single male (a5)	0.291	1.053	1.043	1.025	1.015	0.972
Married male	0.360	1.072	1.062	1.043	1.047	1.022
Married male (a1)	0.305	1.072	1.055	1.030	1.111	1.054
Married male (a2)	0.321	1.071	1.060	1.043	1.134	1.100
Married male (a3)	0.332	1.063	1.054	1.039	1.143	1.120
Married male (a4)	0.340	1.064	1.055	1.041	1.153	1.134
Married male (a5)	0.346	1.062	1.055	1.042	1.163	1.149
Female	0.235	1.072	1.056	1.042	1.029	0.979
Single female	0.251	1.045	1.028	1.032	0.948	0.822
Single female (a1)	0.284	1.064	1.048	1.034	0.724	0.511
Single female (a2)	0.300	1.065	1.053	1.041	0.750	0.637
Single female (a3)	0.310	1.059	1.047	1.039	0.774	0.686
Single female (a4)	0.317	1.060	1.050	1.036	0.800	0.729
Single female (a5)	0.324	1.058	1.050	1.036	0.823	0.761
Married female	0.224	1.092	1.076	1.050	1.086	1.092
Married female (a1)	0.247	1.076	1.056	1.035	0.674	0.693
Married female (a2)	0.267	1.074	1.059	1.041	0.792	0.798
Married female (a3)	0.284	1.066	1.054	1.041	0.866	0.868
Married female (a4)	0.298	1.066	1.055	1.041	0.929	0.930
Married female (a5)	0.312	1.064	1.056	1.042	0.986	0.986

[22] Flat Tax (0.0) is flat tax and 0 deduction, Flat Tax (0.2) is flat tax with 0.2 deduction, Flat Tax (0.4) is flat tax with 0.4 deduction

[23] NIT (1): $\widehat{y} = 0.5$ for singles, $\widehat{y} = 1.0$ for married, $B = 0.05$ for singles, $B = 0.10$ for married, s=0.1

[24] NIT (2): $\widehat{y} = 1.0$ for singles, $\widehat{y} = 1.0$ for married, $B = 0.10$ for singles, $B = 0.10$ for married, s=0.1

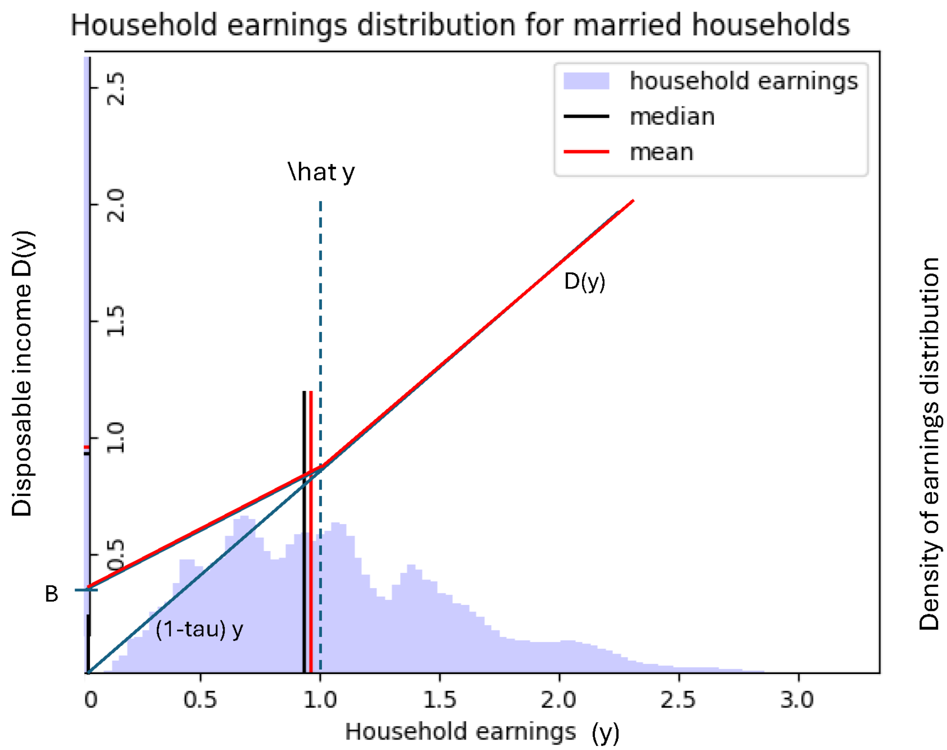


Figure 14: Mapping of labor earnings y into disposable labor earnings $D(y)$ prior to age 65 (solid upper curve) and the earnings distribution of married couples with both spouses being of the lowest ability (colored area)

The break-even labor income $\hat{y}$, at which a worker is no longer a beneficiary of the negative income tax, equals $\hat{y} = B/s$. Figure 14 illustrates the mapping from a household's labor earnings y into its disposable labor income $D(y)$, given a negative income tax policy (B, s). We have superimposed the mapping on the earnings distribution of married couples in which both spouses have lowest abilities and there is a flat tax and zero deduction. These individuals belong to groups with negative consumption equivalents in Table 6 whose welfare we seek to raise with the help of a negative income tax policy.

5.3 Universal basic income

6 Conclusion

The Conclusion comes here.

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A Appendix

A.1 Definition of a Recursive Competitive Equilibrium

We call an equilibrium of the growth adjusted economy a stationary equilibrium.[25] Let $\Phi^M(k^z, e^m, e^w, u^m, u^w, a^m, a^w, F^{Mm}, F^{Mw}, j)$ be the measure of married households with the corresponding characteristics and $\Phi^S(k^z, e, u, a, \iota, F_S^\iota, j)$ be the measure of single households. We now define such a stationary recursive competitive equilibrium as follows:

Definition:

1. The value functions $V^M(\Phi^M)$ and $V^S(\Phi^S)$ and policy functions, $c^z(\Phi^M)$, $k^z(\Phi^M)$, $n^m(\Phi^M)$, $n^w(\Phi^M)$, $c(\Phi^S)$, $k(\Phi^S)$, and $n(\Phi^S)$ solve the consumers' optimization problem given the factor prices and initial conditions.

2. Markets clear:

$$K^z + B^z = \int k^z d\Phi^M + \int k^z d\Phi^S$$

[25]the associated BGP can of course trivially be constructed by scaling all appropriate variables by the growth factor Z_t.

$$L^z = \int \left(n^m w^{zm} + n^w w^{zf}\right) d\Phi^M + \int \left(nw^z\right) d\Phi^S$$

$$\int c^z d\Phi^M + \int c^z d\Phi^S + (\mu+\delta)K^z + G^z = \left(K^z\right)^\alpha \left(L^z\right)^{1-\alpha}$$

3. The factor prices satisfy:

$$\begin{aligned} w^z &= (1-\alpha)\left(\frac{K^z}{L^z}\right)^\alpha \\ r &= \alpha\left(\frac{K^z}{L^z}\right)^{\alpha-1} - \delta \end{aligned}$$

4. The government budget balances:

$$\begin{aligned} g^z\left(2\int d\Phi^M + \int d\Phi^S\right) &+ G^z + (r-\mu)B^z \\ &= \int \left(\tau_k r k^z + \tau_c c^z + \tau_l^M \left(\frac{n^m w^{mz} + n^w w^{wz}}{1+\tilde{\tau}_{ss}}\right)\right) d\Phi^M \\ &+ \int \left(\tau_k r k^z + \tau_c c^z + \tau_l^S \left(\frac{nw^z}{1+\tilde{\tau}_{ss}}\right)\right) d\Phi^S \end{aligned}$$

5. The social security system balances:

$$\Psi^z\left(\int_{j\geq 65} d\Phi^M + \int_{j\geq 65} d\Phi^S\right) = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1+\tilde{\tau}_{ss}}\left(\int_{j<65} (n^m w^{mz} + n^w w^{wz}) d\Phi^M + \int_{j<65} nw^z d\Phi^S\right)$$

6. The assets of the dead are uniformly distributed among the living:

$$\Gamma^z\left(\int \omega(j) d\Phi^M + \int \omega(j) d\Phi^S\right) = \int \left(1-\omega(j)\right) k^z d\Phi^M + \int \left(1-\omega(j)\right) k^z d\Phi^S$$

A.2 Tax Function

Given the tax function

$$ya = \theta_0 y^{1-\theta_1}$$

we employ, the after tax income is defined as

$$ya = (1 - \tau(y))y$$

and thus

$$\theta_0 y^{1-\theta_1} = (1 - \tau(y))y$$

and thus

$$\begin{aligned}
1 - \tau(y) &= \theta_0 y^{-\theta_1} \\
\tau(y) &= 1 - \theta_0 y^{-\theta_1} \\
T(y) &= \tau(y)y = y - \theta_0 y^{1-\theta_1} \\
T'(y) &= 1 - (1 - \theta_1)\theta_0 y^{-\theta_1}
\end{aligned}$$

Thus the tax wedge for any two incomes (y_1, y_2) is given by

$$1 - \frac{1 - T'(y_2)}{1 - T'(y_1)} = 1 - \left(\frac{y_2}{y_1}\right)^{-\theta_1} = 1 - \frac{1 - \tau(y_2)}{1 - \tau(y_1)} \tag{14}$$

and therefore independent of the scaling parameter θ_0 [26]. Thus by construction one can raise average taxes by lowering θ_0 and not change the progressivity of the tax code, since (as long as tax progressivity is defined by the tax wedges) the progressivity of the tax code[27] is uniquely determined by the parameter θ_1. Heathcote et al. (2017) estimate the parameter $\theta_1 = 0.18$ for all households. Above we let θ_1 vary by family type.

[26] It should be noted that the last inequality only holds in the absence of additional lumpsum transfers.

[27] Note that

$$1 - \tau(y) = \frac{1 - T'(y)}{1 - \theta_1} > 1 - T'(y)$$

and thus as long as $\theta_1 \in (0, 1)$ we have that

$$T'(y) > \tau(y)$$

and thus marginal tax rates are higher than average tax rates for all income levels.

A.3 Modeling Choices and Comparison to Holter, Krueger and Stepanchuk (2019)

Our model builds on and extends the model in Holter et al. (2019) in several ways. One major extension is that our framework models an extensive margin of labor supply and accumulation of experience for men. Holter et al. (2019) only model an extensive margin of labor supply for women. Furthermore, we allow individuals to continue working beyond age 64, and we introduce an endogenous retirement decision. In our benchmark model, individuals above 64 who are working do not receive social security benefits but they can choose to retire, stop working and receive benefits.

There are some other minor differences between our framework and Holter et al. (2019). Holter et al. (2019) have a distribution of fixed utility costs of working for single and married women. We only have one fixed utility cost of work but let it differ by gender and marital status. Furthermore, there are some differences in the fiscal system and in the choice of calibration targets. We use only one government budget constraint and let the social security benefits be an expense in the overall government budget. Holter et al. (2019) have a balanced social security budget in addition to a balanced government budget. We also provide a more realistic modeling of social security benefits and let them depend on gender, marital status, innate ability and years of experience. In Holter et al. (2019) everyone receives the same fixed social security benefit. Finally, we view innate ability in our model as a proxy for the level of education and we calibrate the correlation of spouses level of education. Holter et al. (2019) instead targeted the spousal correlation of wages.

A.4 Estimation of Returns to Experience and Shock Processes From the PSID

We take the log of equation 3 and estimate a log(wage) equation using data from the non-poverty sample of the PSID 1968-1997. Equation 4 is estimated using the residuals from Equation 3.

To control for selection into the labor market, we use Heckman's 2-step selection model.

For people who are working and for which we observe wages, the wage depends on years of labor market experience, e, as well as dummies for the year of observation, D:

$$log(w_{it}) = \phi_i(constant + D_t'\zeta + \gamma_1 e_{it} + \gamma_2 e_{it}^2 + \gamma_3 e_{it}^3 + u_{it}) \tag{15}$$

Labor market experience is the only observable determinants of wages in the model apart from gender. The probability of participation (or selection equation) depends on various demographic characteristics, Z:

$$\Phi(participation) = \Phi(Z_{it}'\xi + \upsilon_{it}) \tag{16}$$

The variables included in Z are marital status, age, the number of children, years of schooling, time dummies, and an interaction term between years of schooling and age. To obtain the parameters, σ^ι, ρ^ι and σ_{α^ι} we obtain the residuals $u_i t$ and use them to estimate the below equation by fixed effects estimation:

$$u_{it} = \alpha_i + \rho u_{it-1} + \epsilon_{it} \tag{17}$$

The parameters can be found in Table 1.

A.5 Matching of Individuals in Marriage

Single households face an age-dependent probability, $M(j)$, of becoming married, whereas married households face an age-dependent probability, $D(j)$, of divorce. There is assortative matching in the marriage market, in the sense that there is a greater chance of marrying someone with similar ability, a fact that singles rationally foresee.

To implement assortative matching numerically, we introduce the match index, M_n, in the simulation stage of our computational algorithm. M_n is a convex combination of a random shock, $\varsigma \sim U[0,1]$ and permanent ability, a:

$$M_n = (1-\varphi)\varsigma + \varphi a \tag{18}$$

where $\varphi \in [0,1]$. Single men and women matched to get married in this period are sorted, within their gender, based on M_n, and assigned the partner of the opposite gender with the same rank. The parameter, φ, thus determines the degree of assortative matching, based on ability. If $\varphi = 0$, then matching is random and if $\varphi = 1$ spouses will have identical ability.

Singles have rational expectations with respect to potential partners. The matching function in Equation 18 implies conditional probabilities for marrying someone of ability, a', given an individual's own ability, a. Conditional on gender, age and permanent ability, we also keep track of the distribution of singles with respect to assets, labor market experience, female participation costs and idiosyncratic productivity shocks. A single individual can thus have a rational expectation about a potential partner with respect to these characteristics and the expectation will be conditional on the individual's own gender, age and permanent ability.

In section 3 we calibrate the parameter φ to match the correlation of the wages of married couples in the data. We model the normal distributions of abilities, $a \sim N(0, \sigma_a^{\iota 2})$, using Tauchen (1986)'s method and 5 discrete values of a, placed at $\{-1.5\sigma_a^\iota, -0.75\sigma_a^\iota, 0, 0.75\sigma_a^\iota, 1.5\sigma_a^\iota\}$. Given our calibrated value of φ we obtain the below matrix of marriage probabilities across ability levels:

$$\phi^{-\iota}(a|a^\iota;\varphi) = \begin{bmatrix} 0.414 & 0.387 & 0.170 & 0.029 & 0.000 \\ 0.208 & 0.397 & 0.288 & 0.094 & 0.014 \\ 0.069 & 0.220 & 0.434 & 0.214 & 0.063 \\ 0.014 & 0.093 & 0.279 & 0.428 & 0.186 \\ 0.000 & 0.027 & 0.143 & 0.311 & 0.519 \end{bmatrix}$$

A.6 Additional Tables and Figures